

Gruppo Tecniche di Giunzione
Vicenza 27 e 28 Marzo 2008

**LOCAL STRAIN ENERGY DENSITY AND
FATIGUE STRENGTH OF WELDED JOINTS**

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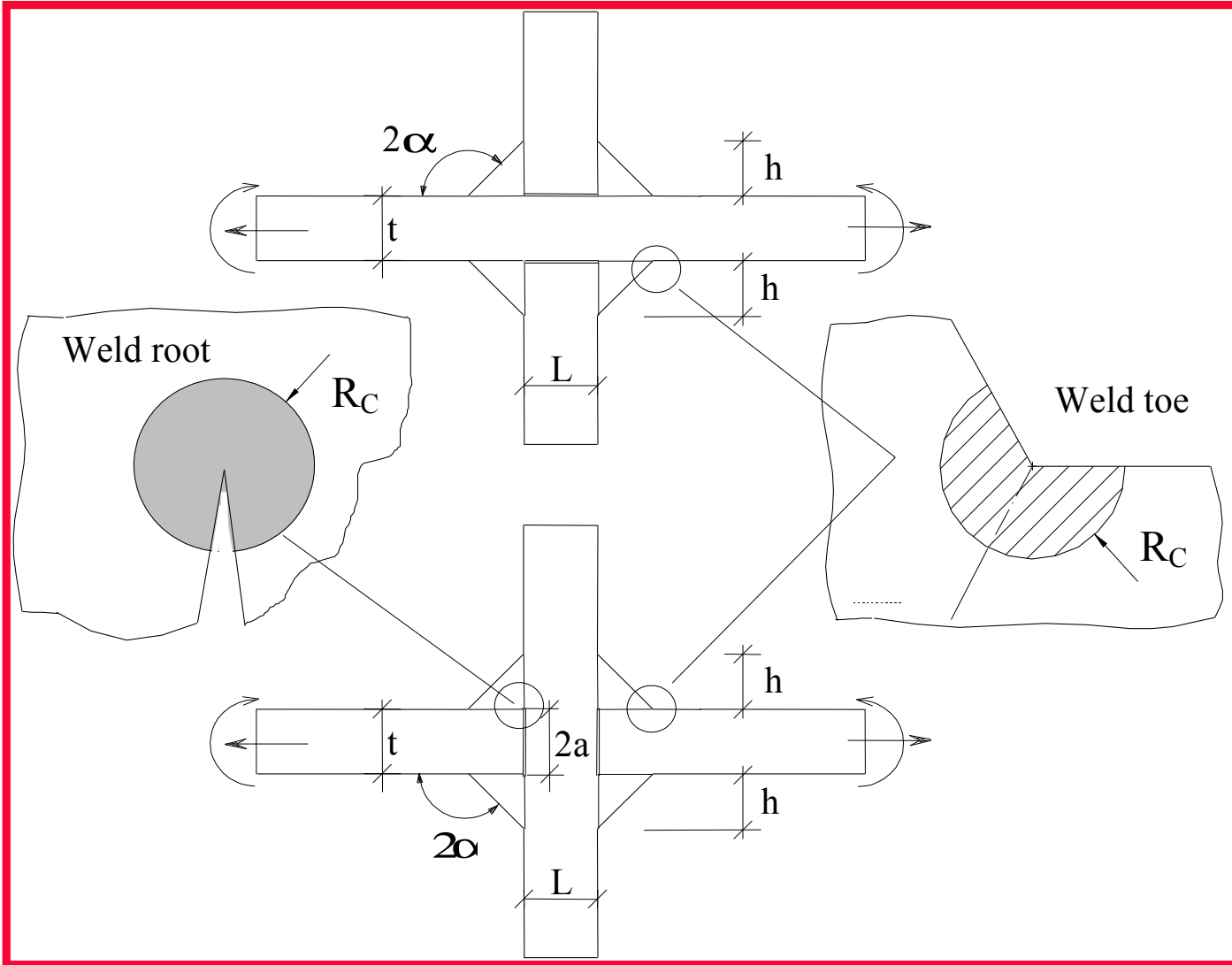
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Vicenza 27 e 28 Marzo 2008

- Weld bead geometry cannot, by its nature, be precisely defined. Parameters such as bead shape and toe radius vary from joint to joint.**
- Conventional welding techniques result in very small values for the toe radius.**
- In the present contribution the weld toe region is modeled as a sharp, zero radius, V-shaped notch.**
- The intensity of asymptotic stress distributions obeying Williams' solution are quantified by means of the Notch Stress Intensity Factors (NSIFs).**

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Vicenza 27 e 28 Marzo 2008



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Vicenza 27 e 28 Marzo 2008

$$K_1^N = \sqrt{2\pi} \lim_{r \rightarrow 0^+} r^{1-\lambda_1} \sigma_{\theta\theta} (r, \theta = 0) \quad K_2^N = \sqrt{2\pi} \lim_{r \rightarrow 0^+} r^{1-\lambda_2} \sigma_{r\theta} (r, \theta = 0)$$

Mode I stress distribution is (Lazzarin and Tovo, 1998):

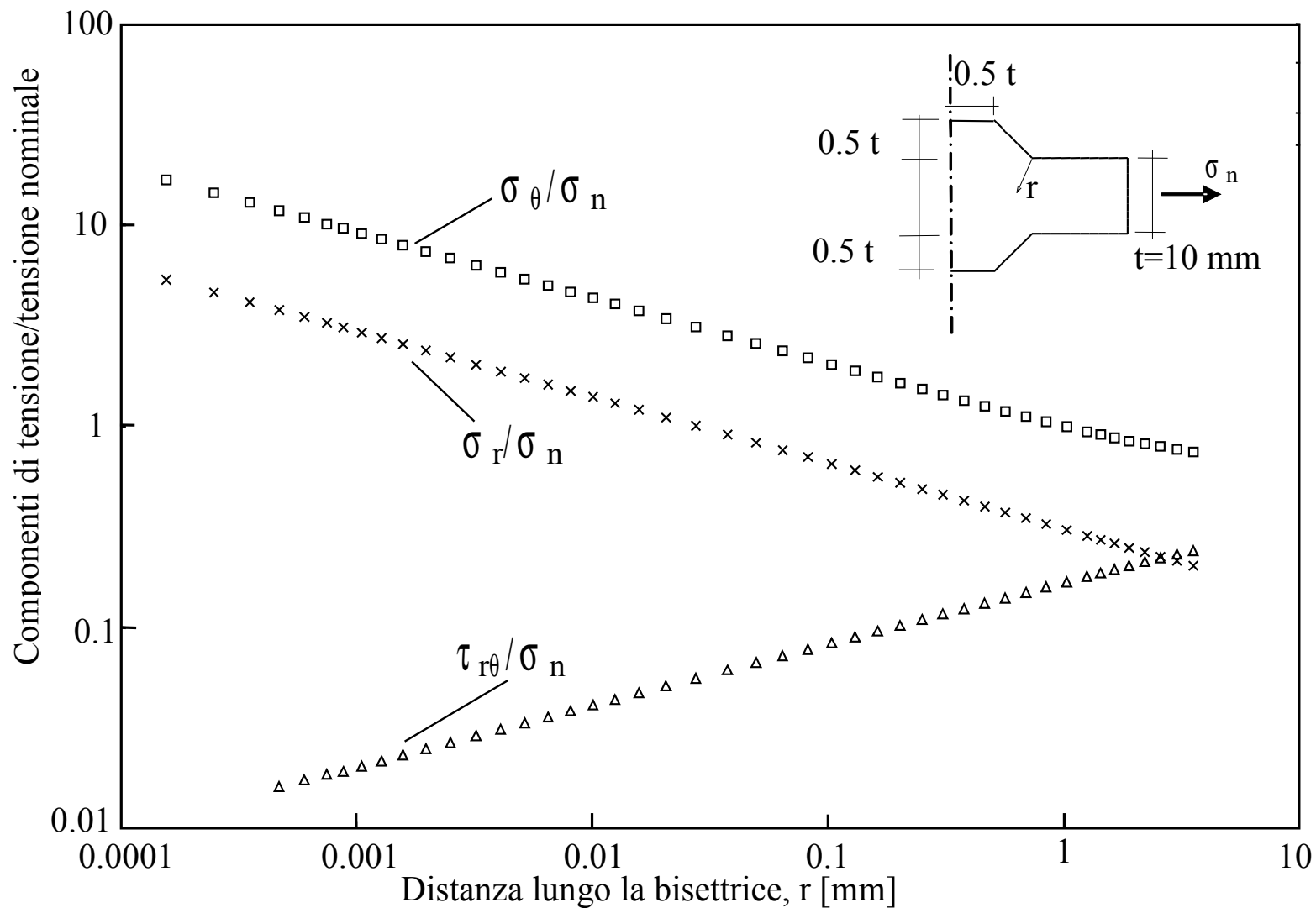
$$\begin{Bmatrix} \sigma_{\theta} \\ \sigma_r \\ \tau_{r\theta} \end{Bmatrix}_{\rho=0} = \frac{1}{\sqrt{2\pi}} \frac{r^{\lambda_1} K_1^N}{(1+\lambda_1) + \chi_1(1-\lambda_1)} \begin{Bmatrix} (1+\lambda_1) \cos(1-\lambda_1)\theta \\ (3-\lambda_1) \cos(1-\lambda_1)\theta \\ (1-\lambda_1) \sin(1-\lambda_1)\theta \end{Bmatrix} + \chi_1(1-\lambda_1) \begin{Bmatrix} \cos(1+\lambda_1)\theta \\ -\cos(1+\lambda_1)\theta \\ \sin(1+\lambda_1)\theta \end{Bmatrix}$$

Mode II stress distribution is:

$$\begin{Bmatrix} \sigma_{\theta} \\ \sigma_r \\ \tau_{r\theta} \end{Bmatrix}_{\rho=0} = \frac{1}{\sqrt{2\pi}} \frac{r^{\lambda_2} K_2^N}{(1-\lambda_2) + \chi_2(1+\lambda_2)} \begin{Bmatrix} -(1+\lambda_2) \sin(1-\lambda_2)\theta \\ -(3-\lambda_2) \sin(1-\lambda_2)\theta \\ (1-\lambda_2) \cos(1-\lambda_2)\theta \end{Bmatrix} + \chi_2(1+\lambda_2) \begin{Bmatrix} -\sin(1+\lambda_2)\theta \\ \sin(1+\lambda_2)\theta \\ \cos(1+\lambda_2)\theta \end{Bmatrix}$$

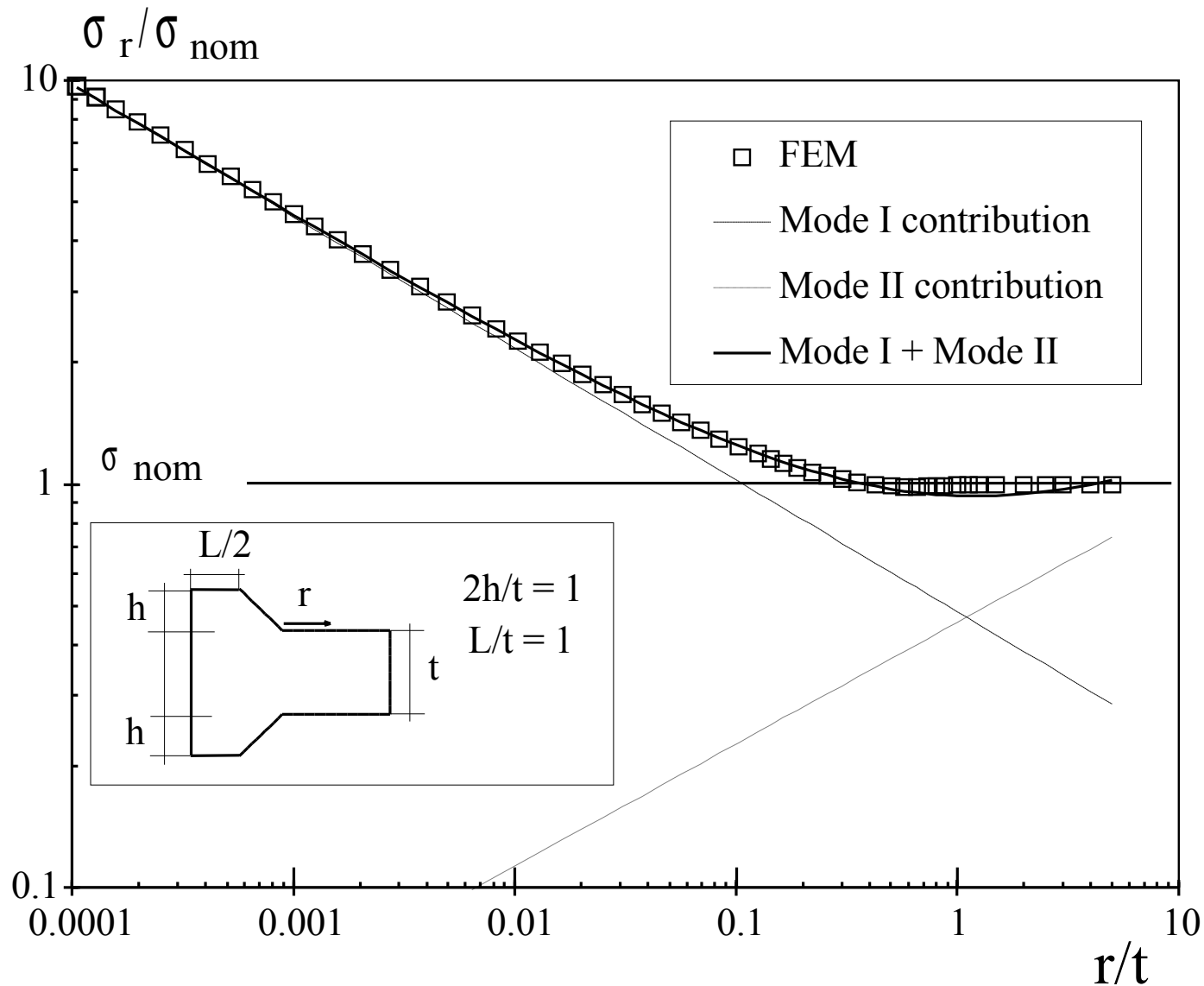
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Vicenza 27 e 28 Marzo 2008



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Vicenza 27 e 28 Marzo 2008

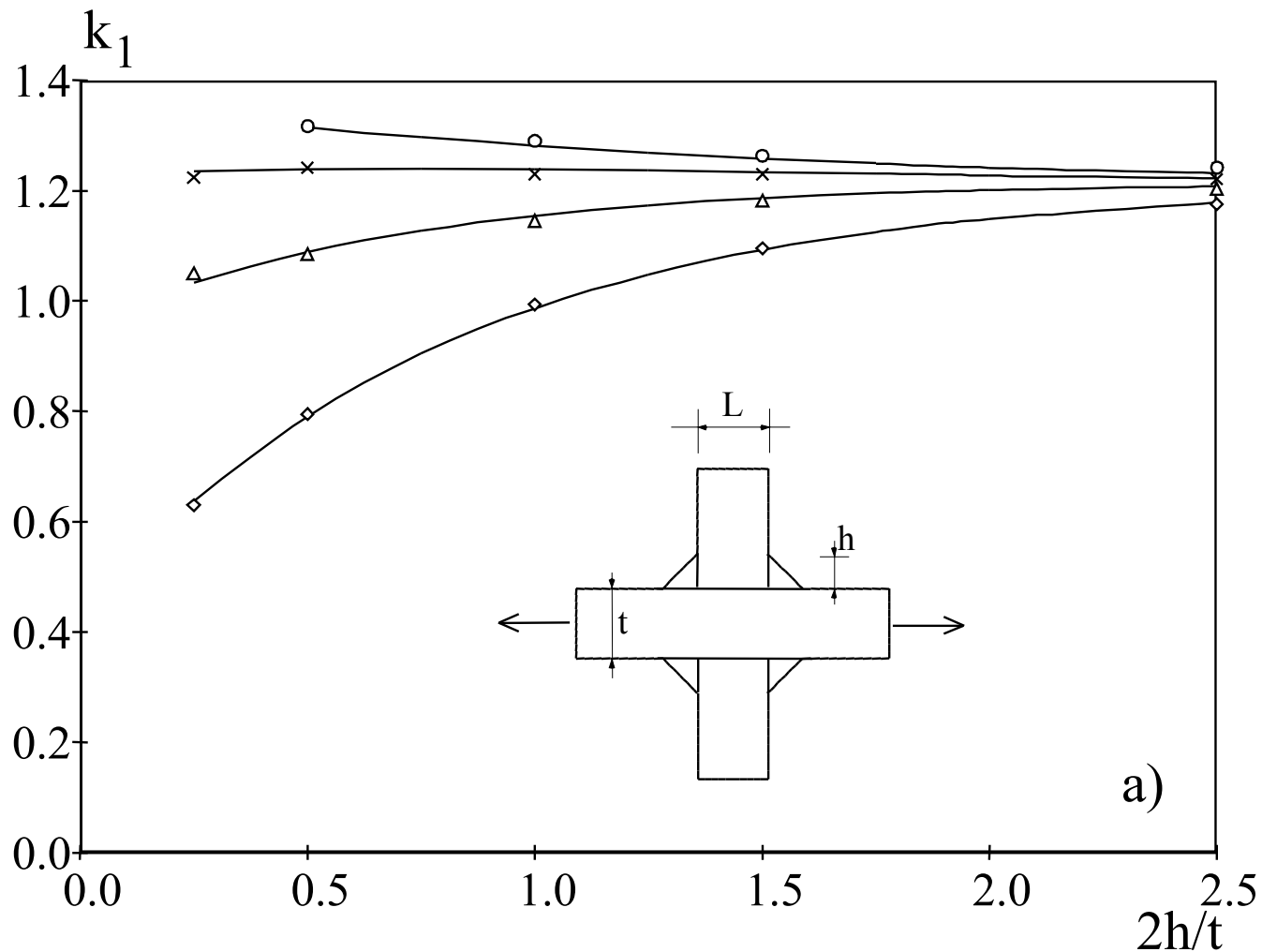


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$$K_1 = k_1 \sigma_0 t^{1-\lambda_1}$$

$$K_2 = k_2 \sigma_0 t^{1-\lambda_2}$$

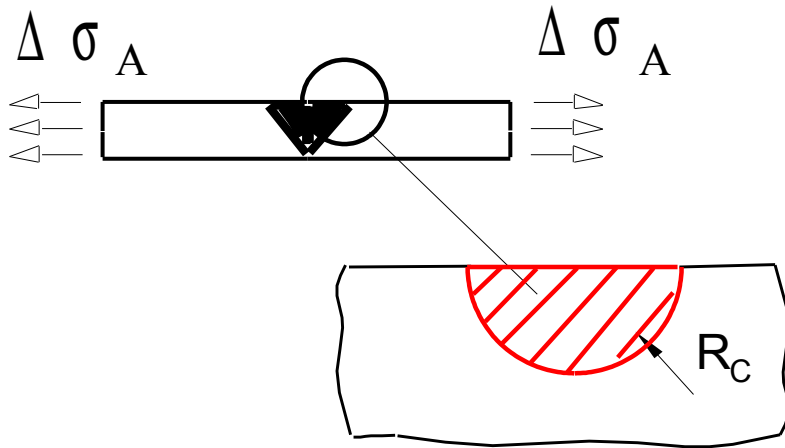


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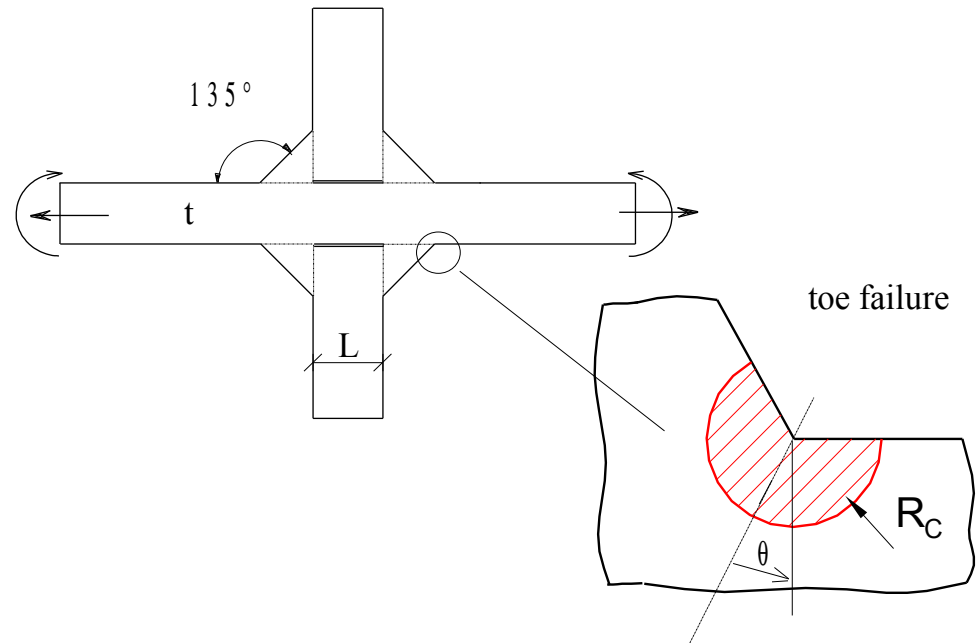
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$2\alpha=180^\circ$

ground butt welded joints



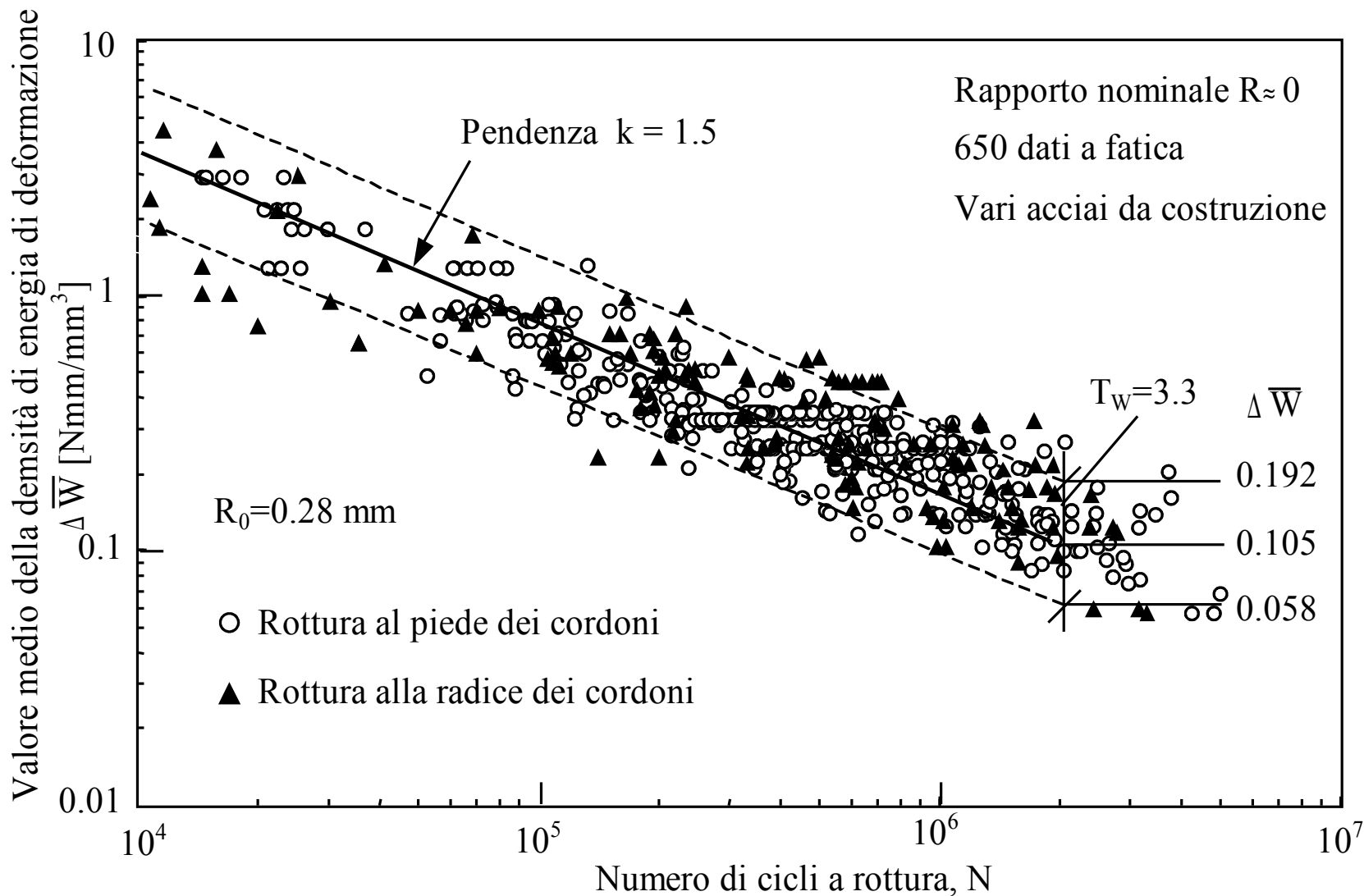
$2\alpha=135^\circ$



$$R_C = \left(\frac{\sqrt{2e_1} \Delta K_{1A}^N}{\Delta \sigma_A} \right)^{\frac{1}{1-\lambda_1}} = 0.28 \text{ mm}$$

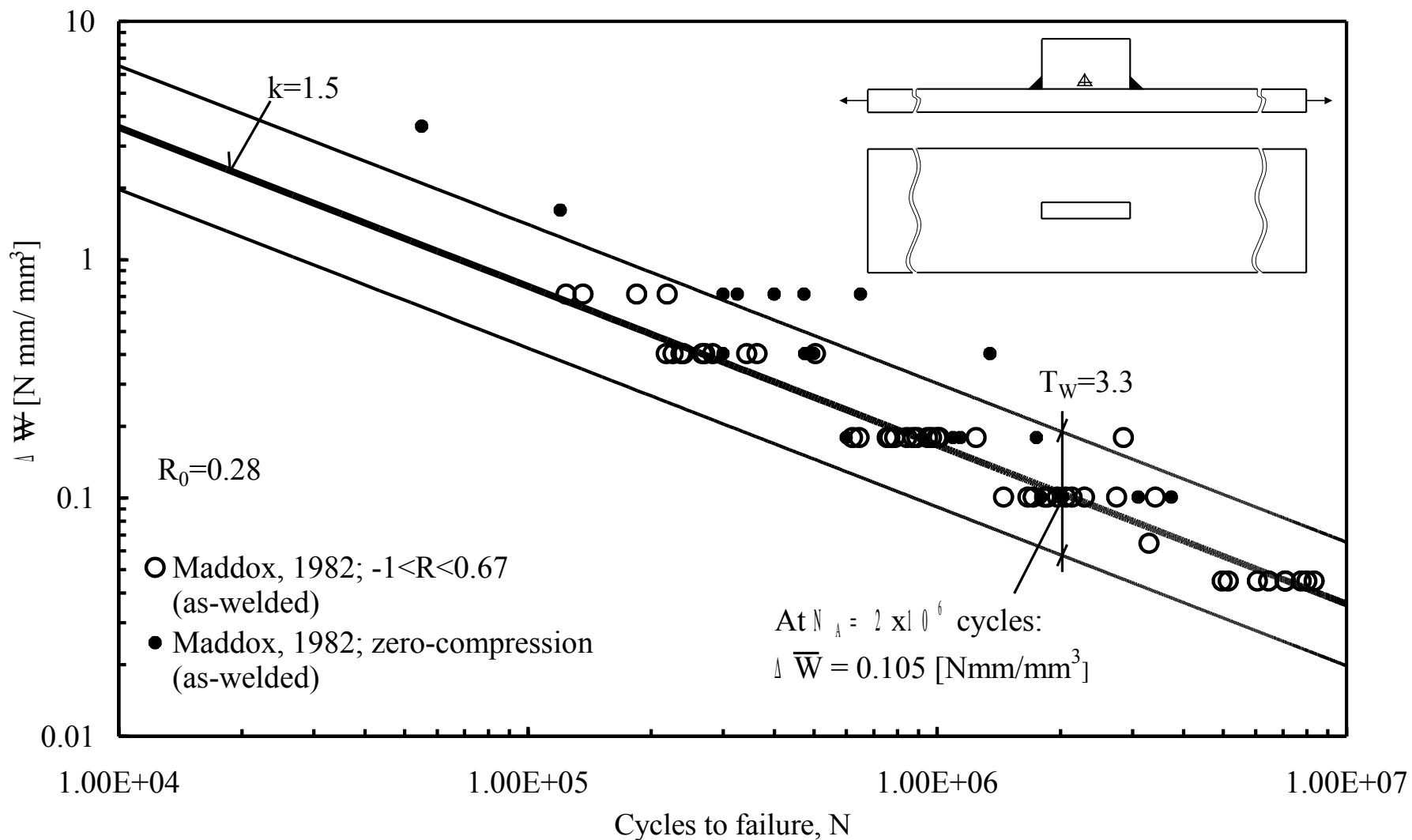
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Vicenza 27 e 28 Marzo 2008




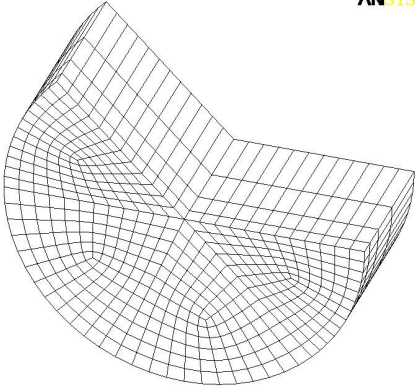

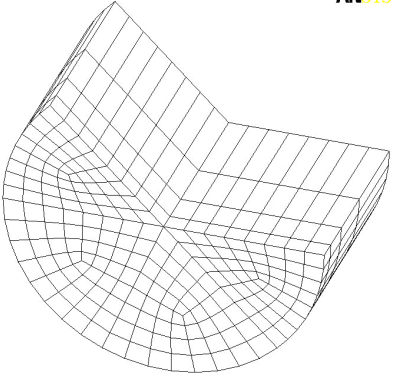

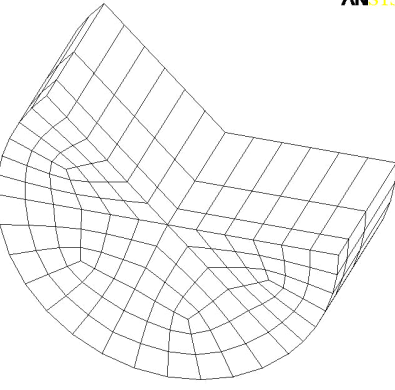

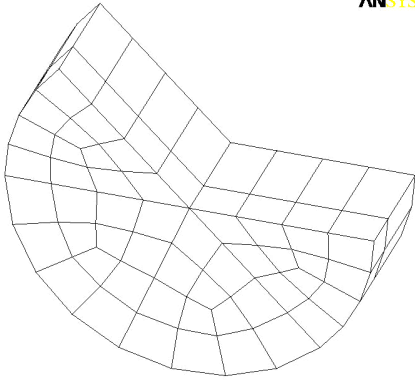

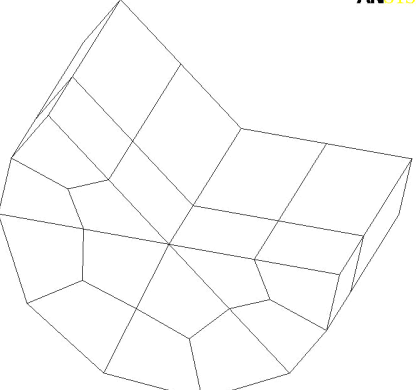

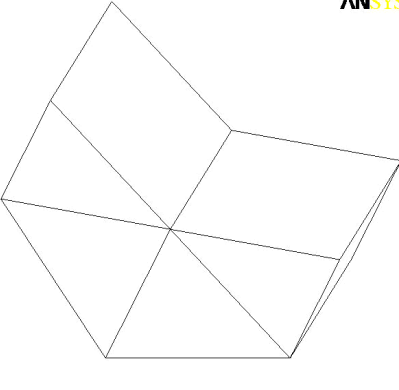
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Gruppo Tecniche di Giunzione

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<p>Mesh</p>   <p>1969 elements</p>	<p>Mesh 2</p>   <p>768 elements</p>	<p>Mesh 3</p>   <p>324 elements</p>
<p>Mesh 4</p>   <p>96 elements</p>	<p>Mesh 5</p>   <p>24 elements</p>	<p>Mesh 6</p>   <p>4 elements</p>

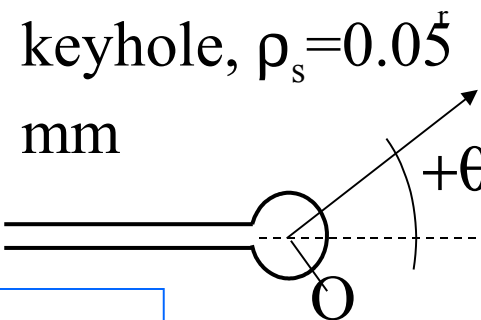
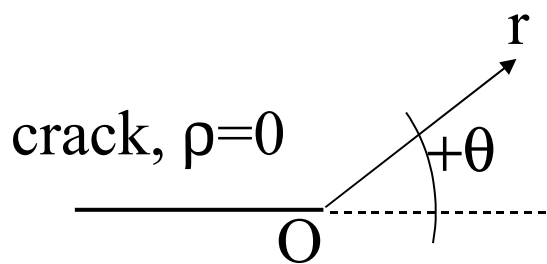
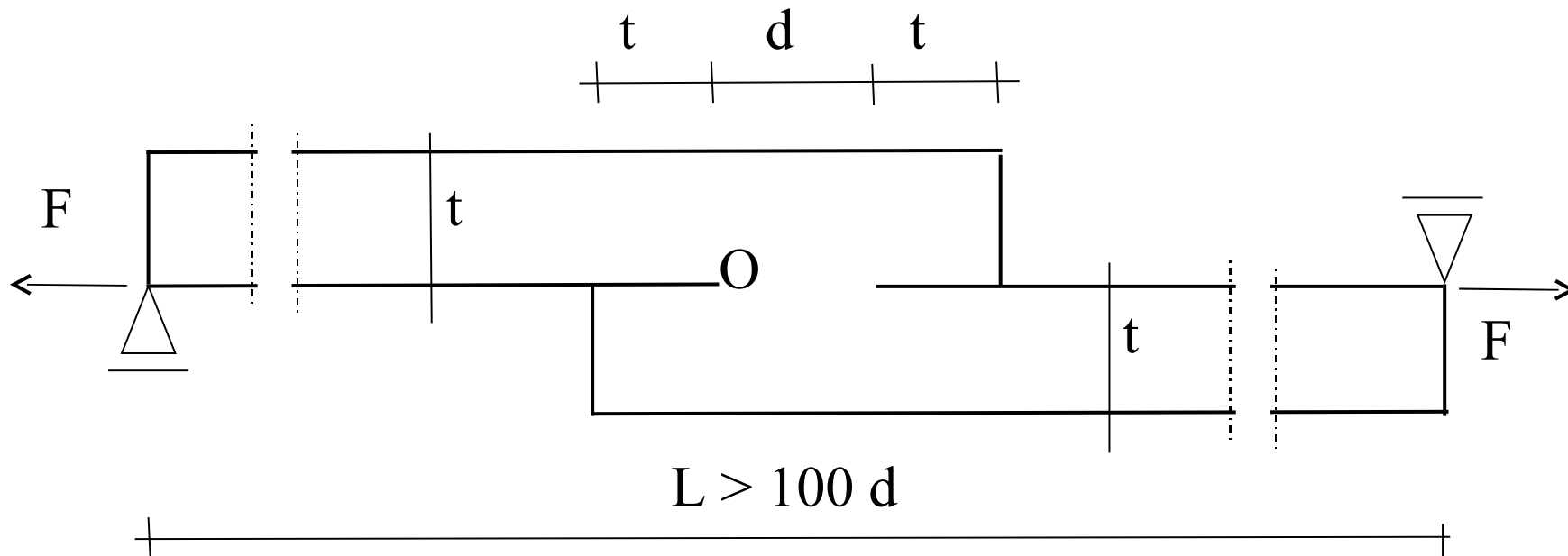
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3D models	Number of FE in the volume	\bar{W} [Nmm/mm ³]	K_1 [MPa mm ^{0.326}]	Δ %
1	1696	0.07937	373.5	0
2	768	0.07903	372.7	0.21
3	324	0.07896	372.5	0.26
4	96	0.07895	372.5	0.26
5	24	0.07790	370.0	0.93
6	4	0.07594	365.3	2.18

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Giunti a semplice sovrapposizione

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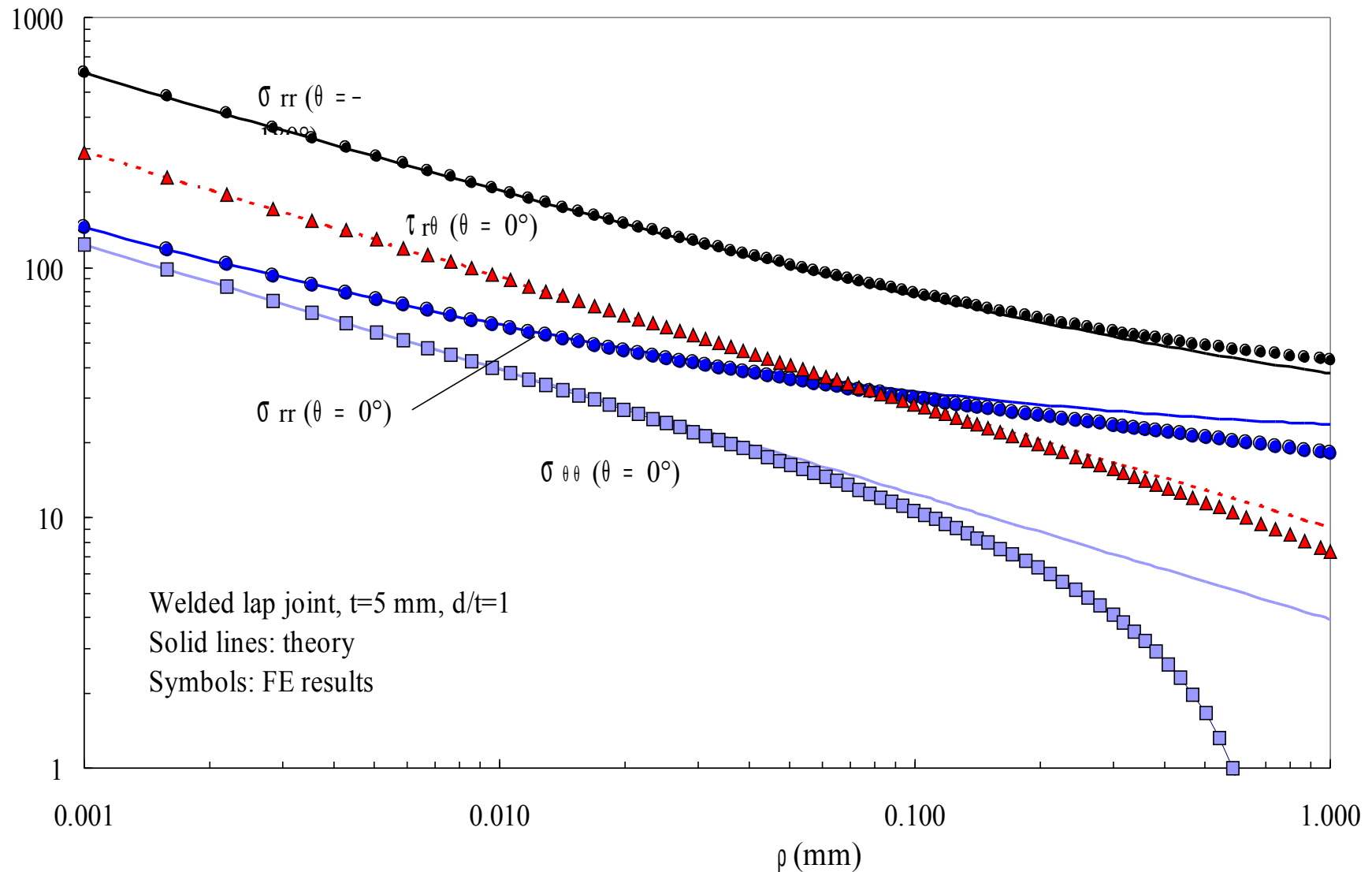
$$\begin{Bmatrix} \sigma_{\theta} \\ \sigma_r \\ \tau_{r\theta} \end{Bmatrix}_{\rho=0}^{(1)} = \frac{1}{\sqrt{2\pi r}} \frac{K_1}{4} \left[\begin{pmatrix} 3 \cos \theta / 2 \\ 5 \cos \theta / 2 \\ \sin \theta / 2 \end{pmatrix} + \begin{pmatrix} \cos 3\theta / 2 \\ -\cos 3\theta / 2 \\ \sin 3\theta / 2 \end{pmatrix} \right] + T \begin{pmatrix} \sin^2 \theta \\ \cos^2 \theta \\ -\sin \theta \cos \theta \end{pmatrix}$$

$$\begin{Bmatrix} \sigma_{\theta} \\ \sigma_r \\ \tau_{r\theta} \end{Bmatrix}_{\rho=0}^{(2)} = \frac{1}{\sqrt{2\pi r}} \frac{K_2}{4} \left[\begin{pmatrix} -3 \sin \theta / 2 \\ -5 \sin \theta / 2 \\ \cos \theta / 2 \end{pmatrix} + 3 \begin{pmatrix} -\sin 3\theta / 2 \\ \sin 3\theta / 2 \\ \cos 3\theta / 2 \end{pmatrix} \right]$$

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Vicenza 27 e 28 Marzo 2008

stresses (MPa)



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Vicenza 27 e 28 Marzo 2008

Kullmer, 1992

$$\sigma_r = \frac{K_{I\rho}}{4\sqrt{2\pi r}} \left\{ \left[5 - 2\frac{\rho}{r} - 3\left(\frac{\rho}{r}\right)^2 \right] \cos\frac{\theta}{2} - \left[1 - \frac{7\rho}{2r} + \frac{5}{2}\left(\frac{\rho}{r}\right)^3 \right] \cos\frac{3\theta}{2} \right\}$$

$$\sigma_\theta = \frac{K_{I\rho}}{4\sqrt{2\pi r}} \left\{ \left[3 + 2\frac{\rho}{r} + 3\left(\frac{\rho}{r}\right)^2 \right] \cos\frac{\theta}{2} + \left[1 + \frac{1\rho}{2r} + \frac{5}{2}\left(\frac{\rho}{r}\right)^3 \right] \cos\frac{3\theta}{2} \right\}$$

$$\tau_{r\theta} = \frac{K_{I\rho}}{4\sqrt{2\pi r}} \left\{ \left[1 + 2\frac{\rho}{r} - 3\left(\frac{\rho}{r}\right)^2 \right] \sin\frac{\theta}{2} + \left[1 + \frac{3\rho}{2r} - \frac{5}{2}\left(\frac{\rho}{r}\right)^3 \right] \sin\frac{3\theta}{2} \right\}$$

$$\sigma_{\max} = \sigma_\theta (\theta = 0^\circ; r = \rho) = 2.1213 \frac{K_{I\rho}}{\sqrt{\pi\rho}}$$

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Vicenza 27 e 28 Marzo 2008

Kullmer, 1992

$$\sigma_r = \frac{-K_{II\rho}}{4\sqrt{2\pi r}} = \left\{ \left[5 - 2\frac{\rho}{r} - 3\left(\frac{\rho}{r}\right)^2 \right] \sin\frac{\theta}{2} - \left[3 - \frac{21\rho}{2r} + \frac{15}{2}\left(\frac{\rho}{r}\right)^3 \right] \sin\frac{3\theta}{2} \right\}$$

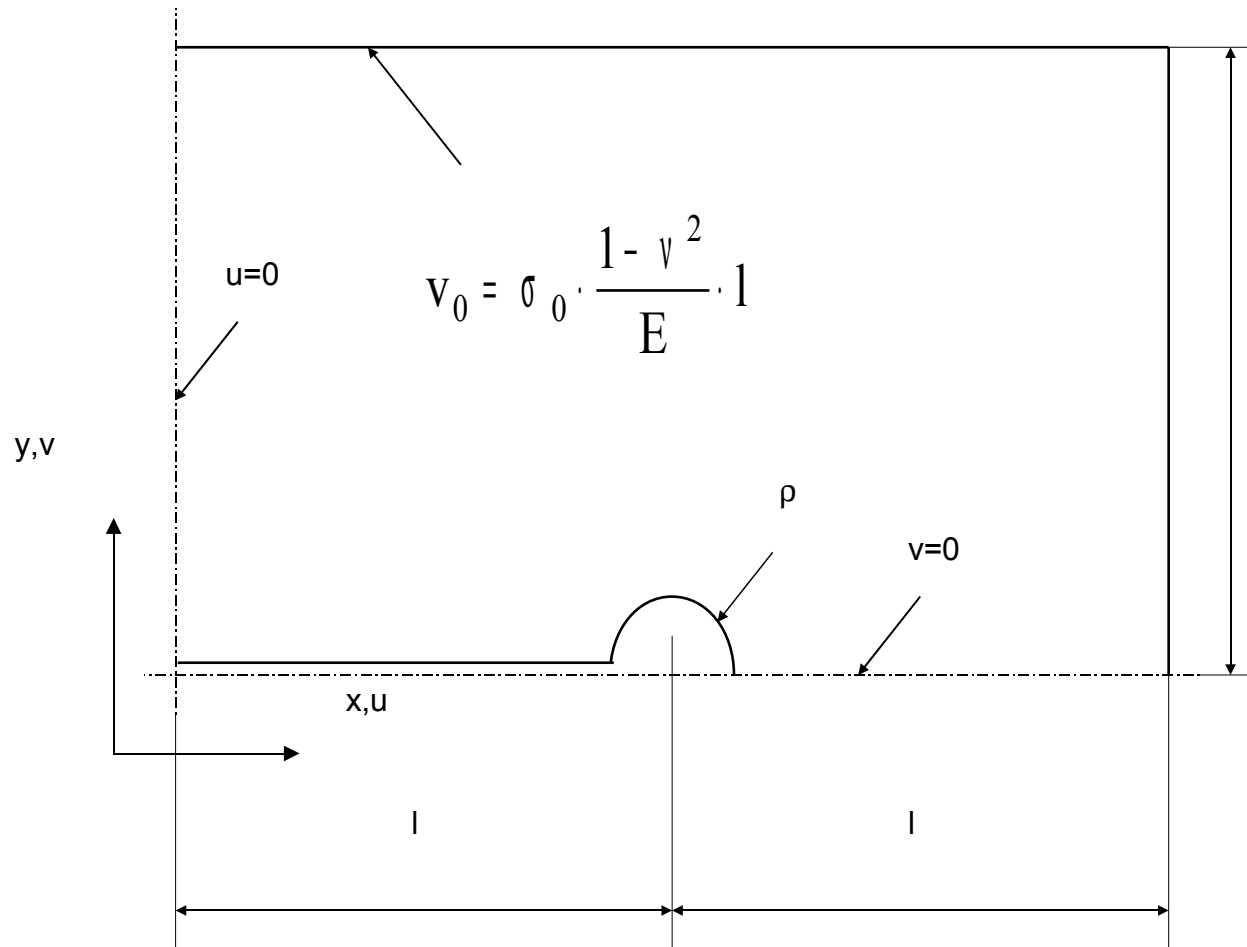
$$\sigma_\theta = \frac{-K_{II\rho}}{4\sqrt{2\pi r}} = \left\{ \left[3 + 2\frac{\rho}{r} + 3\left(\frac{\rho}{r}\right)^2 \right] \sin\frac{\theta}{2} + \left[3 + \frac{3\rho}{2r} + \frac{15}{2}\left(\frac{\rho}{r}\right)^3 \right] \sin\frac{3\theta}{2} \right\}$$

$$\tau_{r\theta} = \frac{K_{II\rho}}{4\sqrt{2\pi r}} = \left\{ \left[1 + 2\frac{\rho}{r} - 3\left(\frac{\rho}{r}\right)^2 \right] \cos\frac{\theta}{2} + \left[3 + \frac{9\rho}{2r} - \frac{15}{2}\left(\frac{\rho}{r}\right)^3 \right] \cos\frac{3\theta}{2} \right\}$$

$$\tau_{\max} = \tau_{r\theta} (\theta = 0^\circ; r = 1.723\rho) = 0.7132 \frac{K_{II\rho}}{\sqrt{\pi\rho}}$$

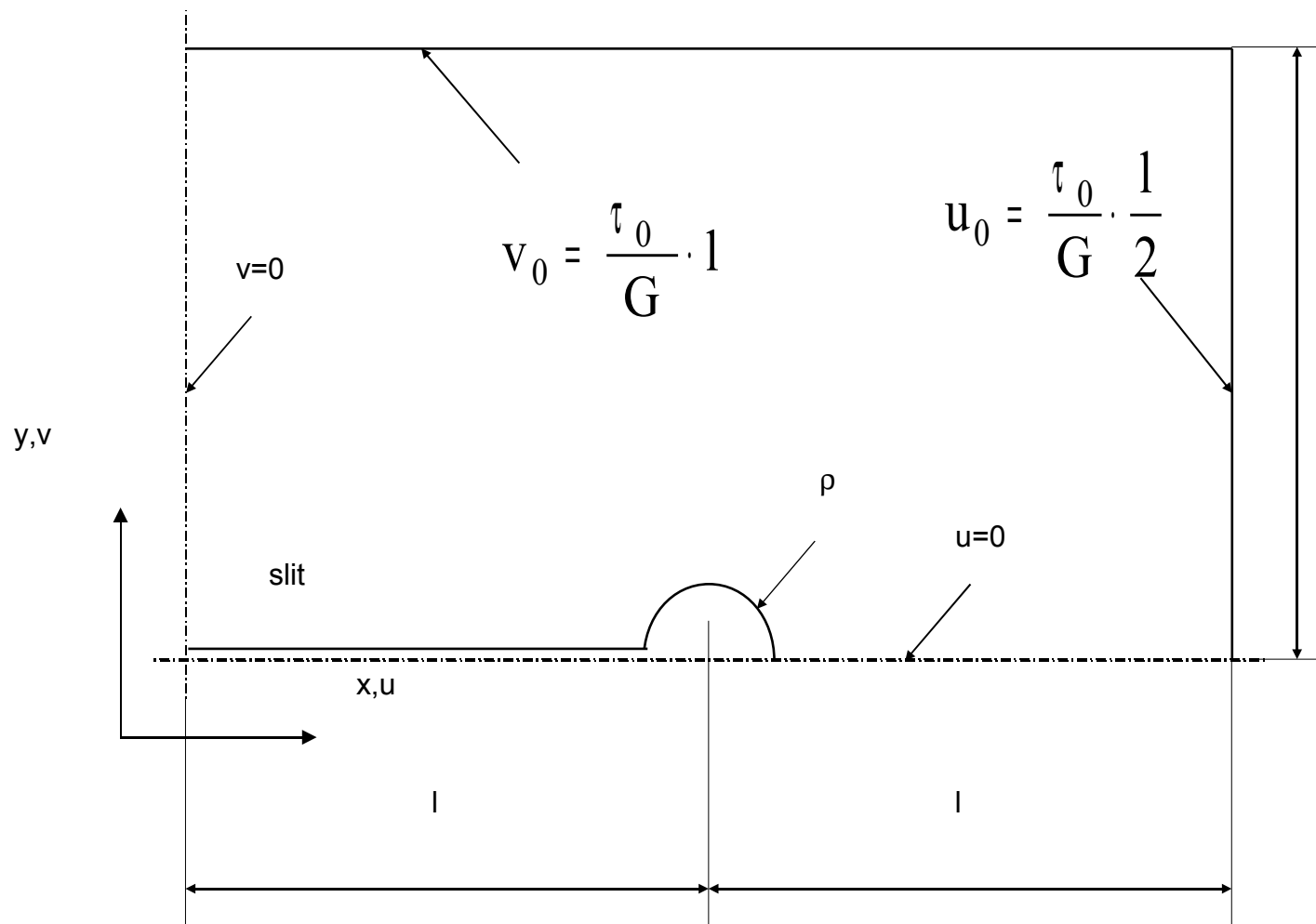
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Vicenza 27 e 28 Marzo 2008



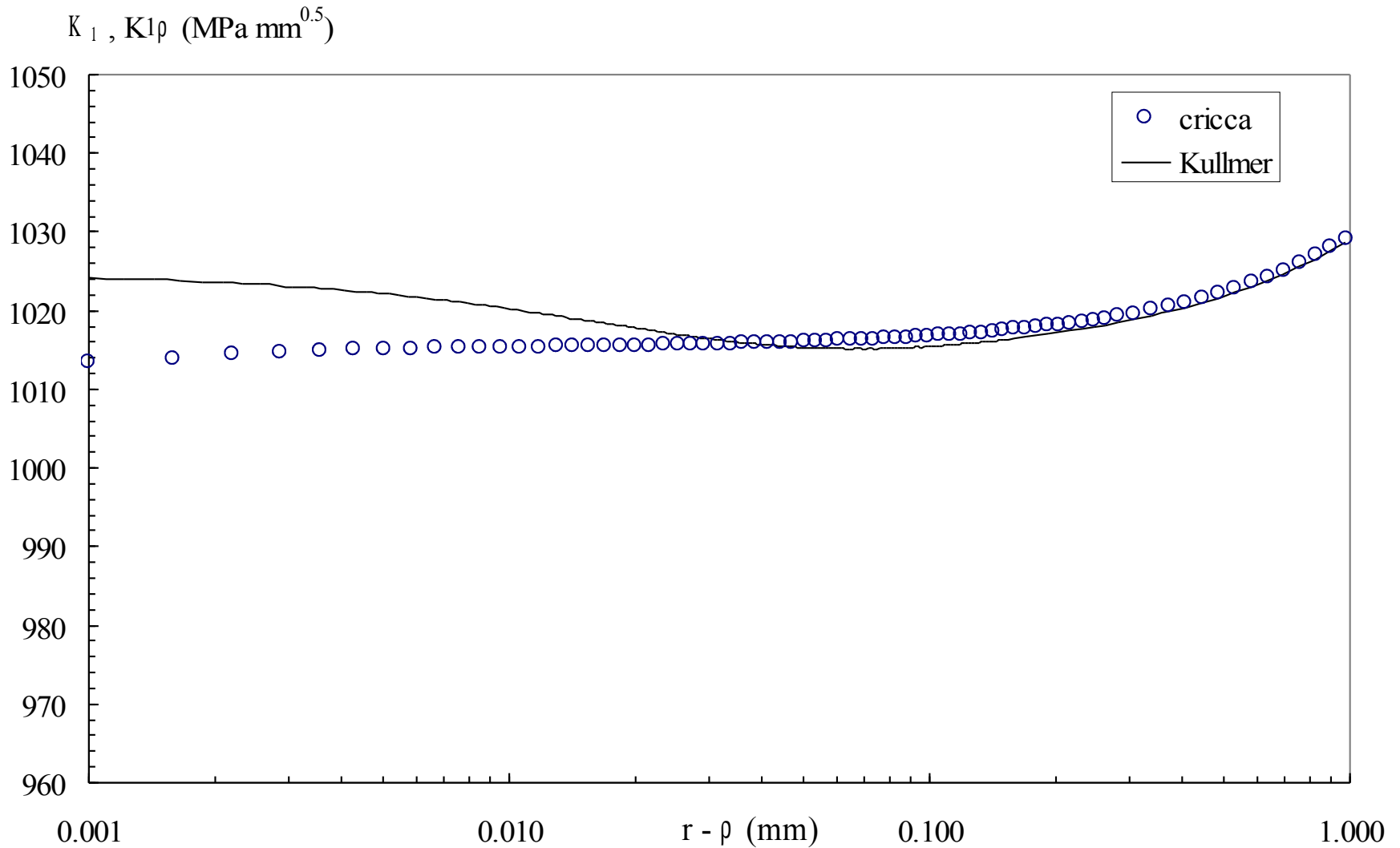
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Vicenza 27 e 28 Marzo 2008



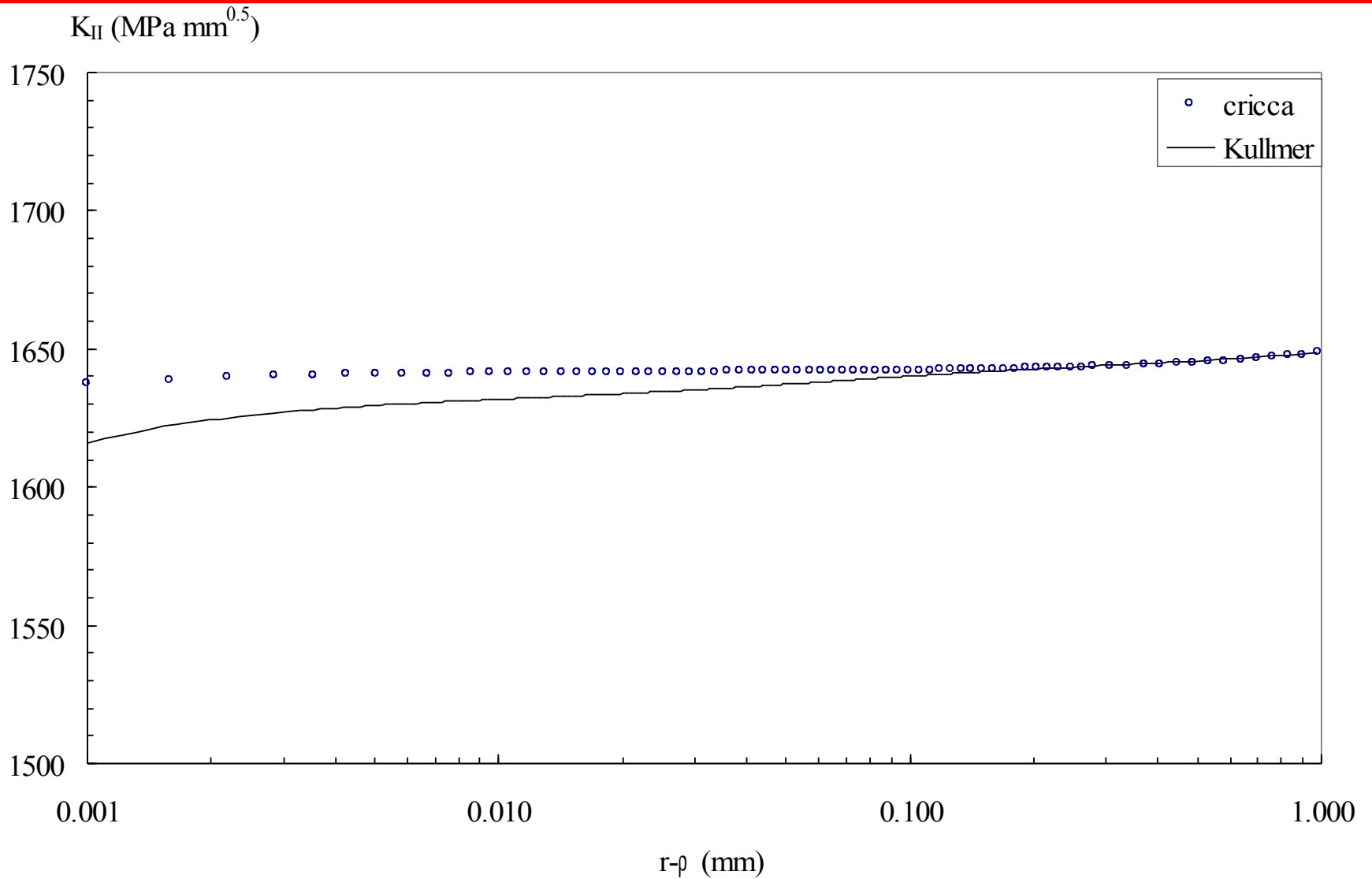
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Vicenza 27 e 28 Marzo 2008



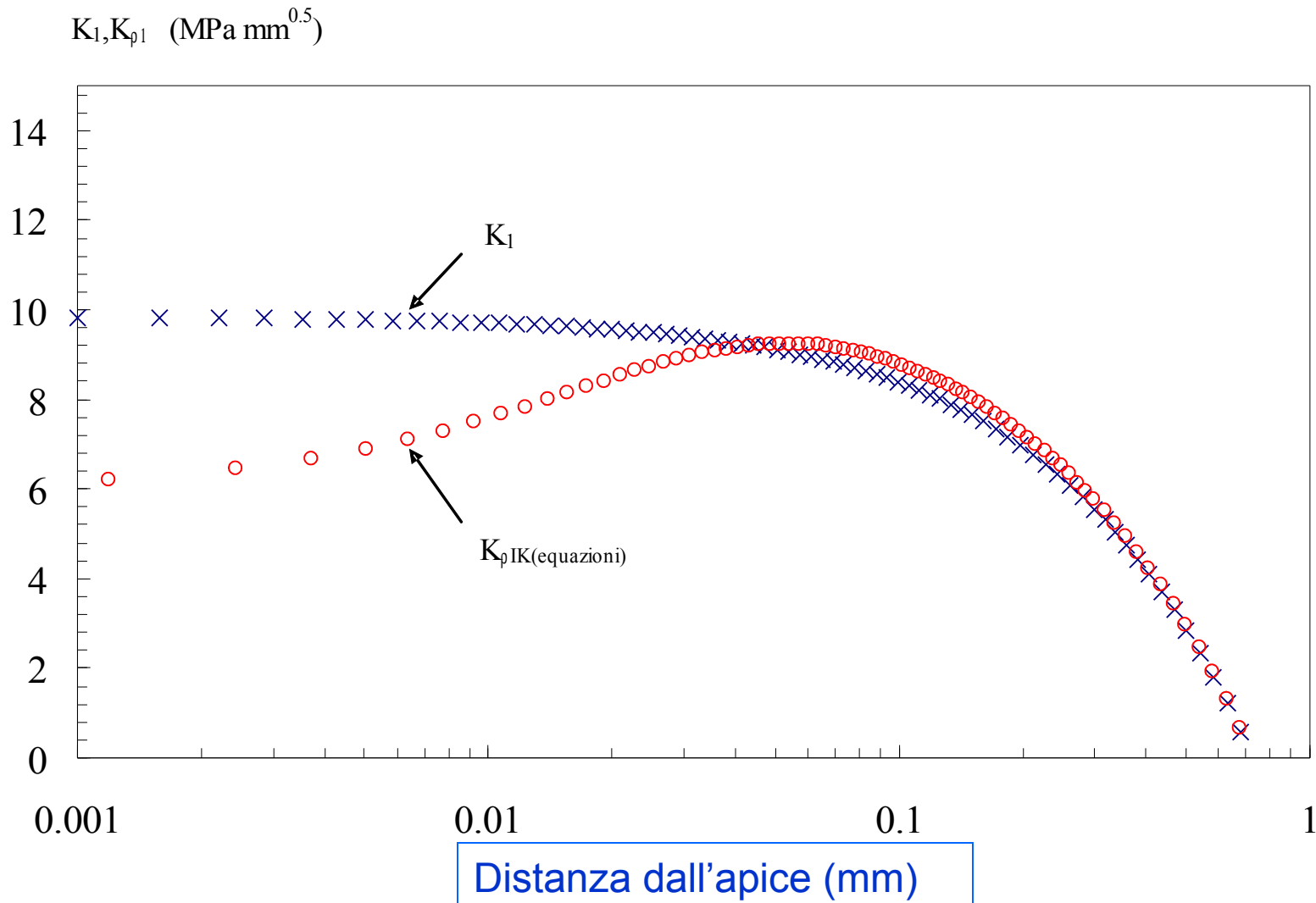
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Vicenza 27 e 28 Marzo 2008



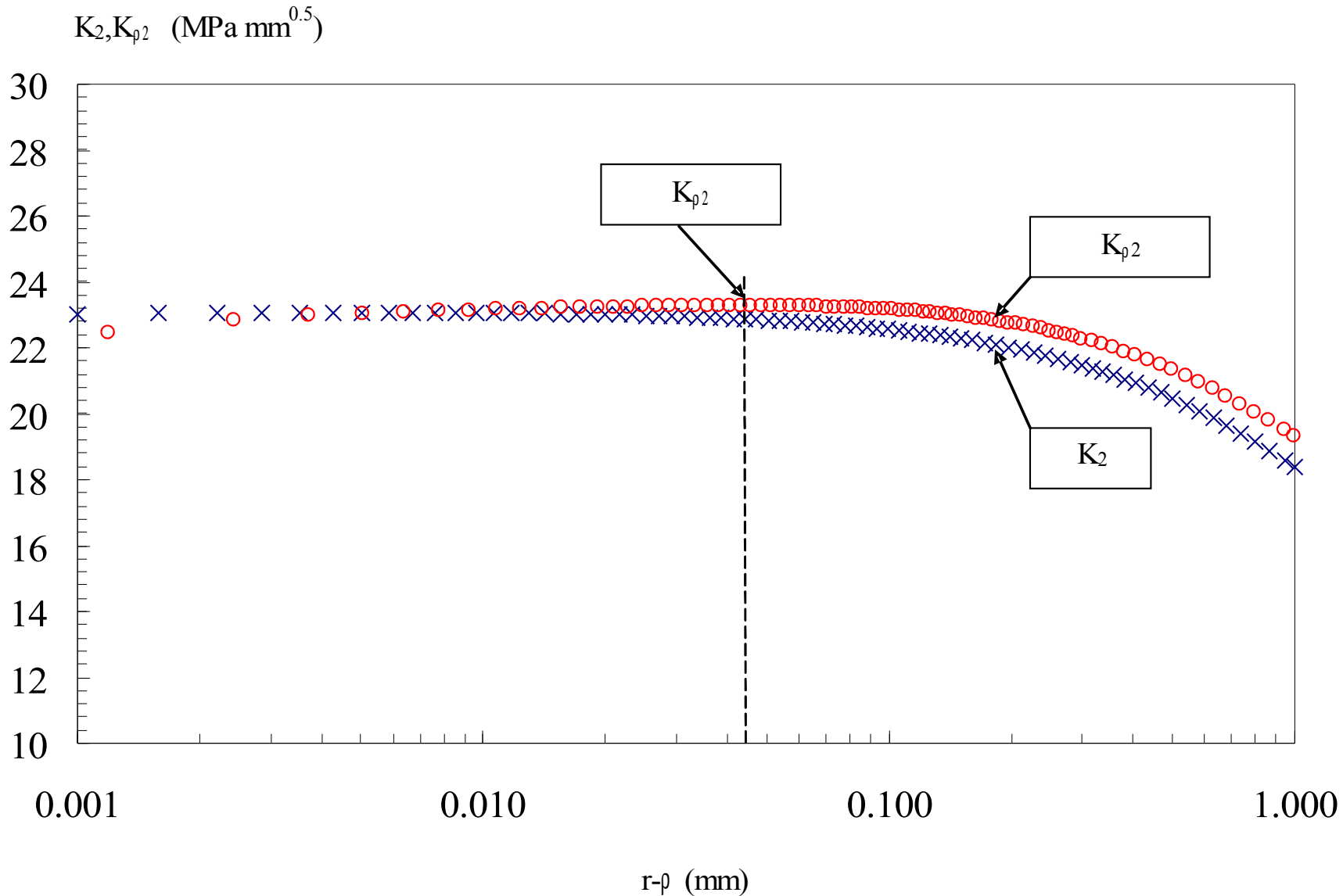
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Vicenza 27 e 28 Marzo 2008



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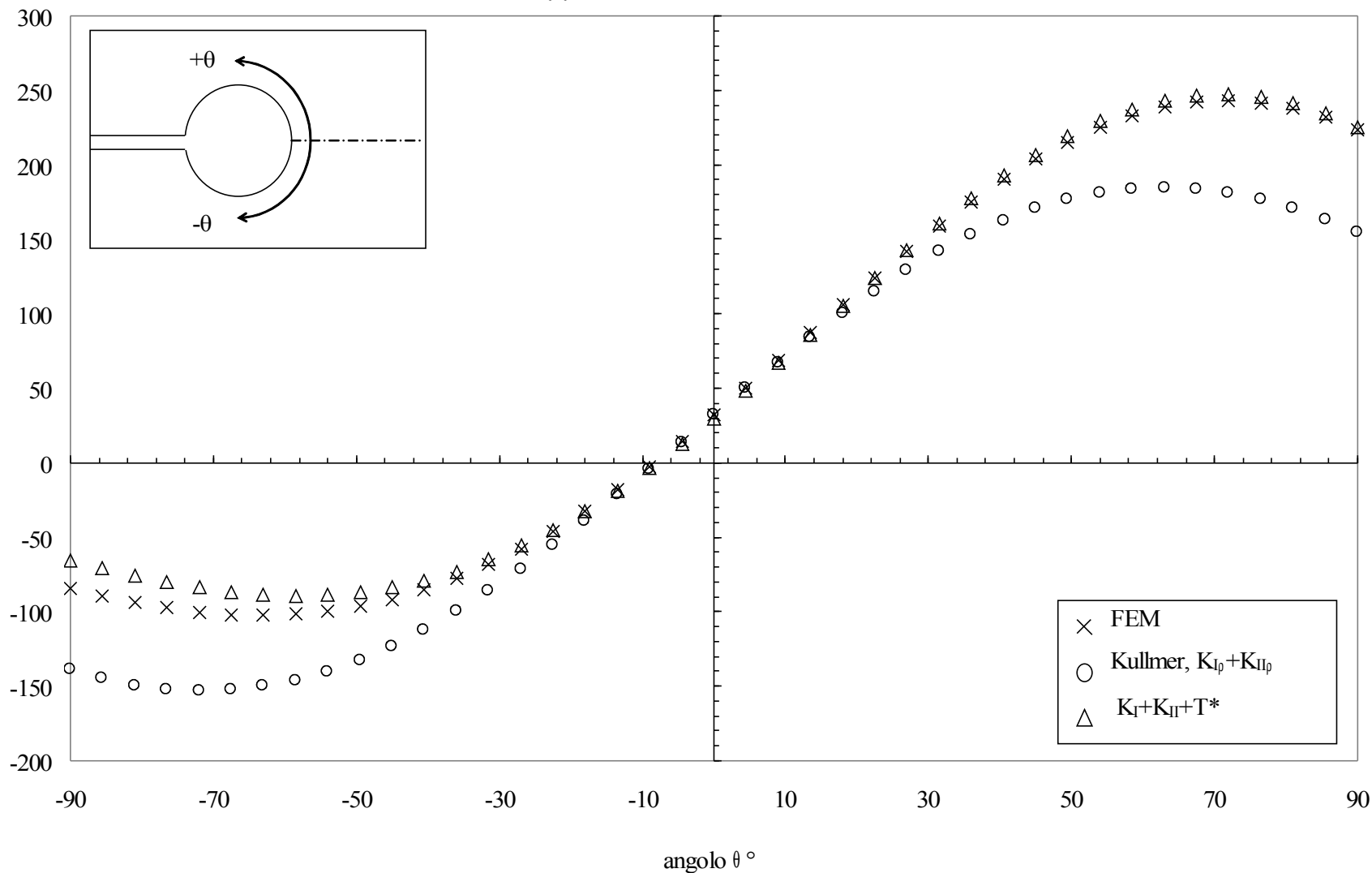
Vicenza 27 e 28 Marzo 2008



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Vicenza 27 e 28 Marzo 2008

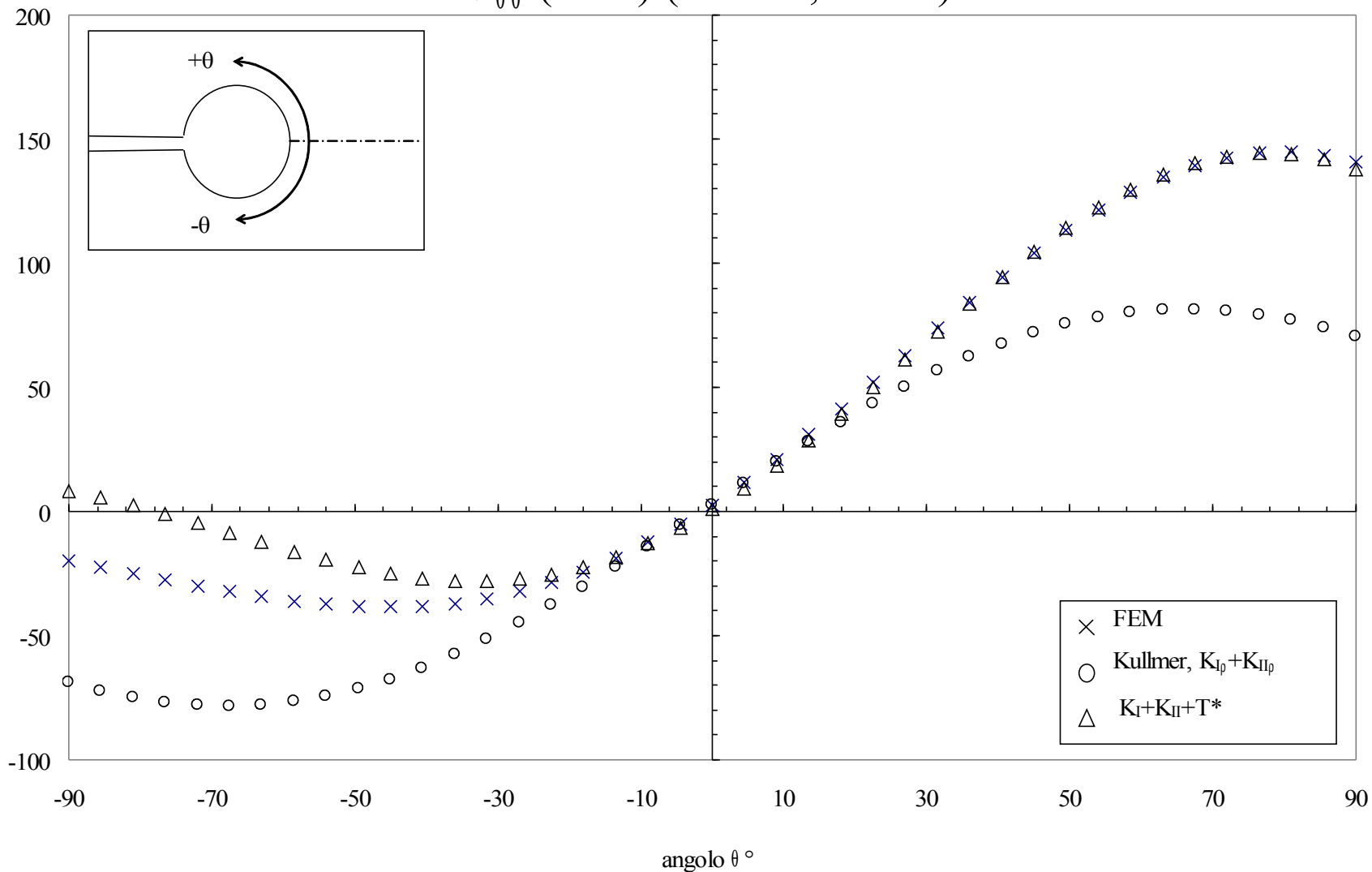
$\sigma_{\theta\theta}$ (MPa) ($t=5\text{mm}$, d/t)



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$\sigma_{\theta\theta}$ (MPa) ($t=1\text{mm}$, $d/t=1$)



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Vicenza 27 e 28 Marzo 2008

	$\rho = 0$		$\rho = 0.05 \text{ mm}$	
	K_I (MPa $\sqrt{\text{mm}}$)	K_{II} (MPa $\sqrt{\text{mm}}$)	$K_{1\rho}$ (MPa $\sqrt{\text{mm}}$)	$K_{2\rho}$ (MPa $\sqrt{\text{mm}}$)
t=1mm, d/t				
0.5	1.62	-12.22	-2.18	-17.05
1	4.35	-10.28	0.52	-11.06
2	8.08	-9.95	5.18	-10.05
3	8.61	-9.95	6.02	-10.05
t=5mm, d/t				
0.5	3.67	-27.30	-0.30	-28.73
1	9.80	-23.02	5.94	-23.25
2	18.14	-22.27	14.86	-22.30
3	19.33	-22.27	16.21	-22.29

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Vicenza 27 e 28 Marzo 2008

		$\rho = 0.05 \text{ mm}$		
		K_t	$K_t^{(b)}$	$K_t^{(a)}$
		FEM	$f(K_I, K_{II})$	$f(K_{1\rho}, K_{2\rho})$
$t=1\text{mm}, d/t$				
	0.5	16.46	15.25	11.77
	1	14.48	14.42	8.14
	2	14.46	14.96	8.78
	3	14.48	15.08	9.04
$t=5\text{mm}, d/t$				
	0.5	26.59	26.38	20.78
	1	24.30	24.73	18.46
	2	25.47	26.29	20.56
	3	25.70	26.60	21.01

Errori del 2-3%,
con l'eccezione del
caso 1 (7%)



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$R_0=0.28\text{mm}$		W/W^*		$(K_t/K_t^*)^2$
$t=1\text{mm}, d/t$	semicircle	circle		
0.5	1.943	1.196		1.292
1	1.788	1.023		1.000
2	1.737	1.002		0.997
3	1.715	1.000		1.000
$t=5\text{mm}, d/t$				
0.5	6.677	4.456		3.372
1	5.636	3.566		2.816
2	5.936	3.804		3.094
3	6.010	3.875		3.150