

- Notch Stress Intensity Factors e SED
- Friction Stir Welding
- Metodi delle distanze critiche
- Metodo del J–integral
- Metodo del gradiente implicito

# IL GRADIENTE IMPLICITO NELLA PREVISIONE DELLA VITA A FATICA NEI GIUNTI SALDATI

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Vicenza

27-28 Marzo 2008

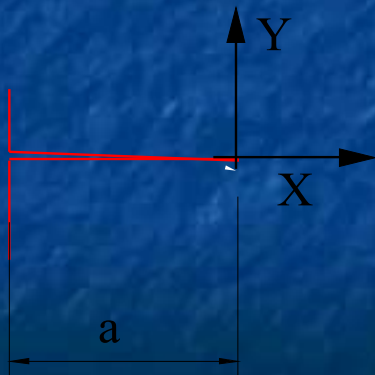


# IL PROBLEMA

## LA VERIFICA STRUTTURALE

- Calcolo: Analisi lineare elastica
- Verifica: Tensioni ammissibili

METODO LOCALE CHE PUO' ESSERE  
ACCETTATO SOLO IN PRESENZA DI  
GRADIENTI NON ELEVATI

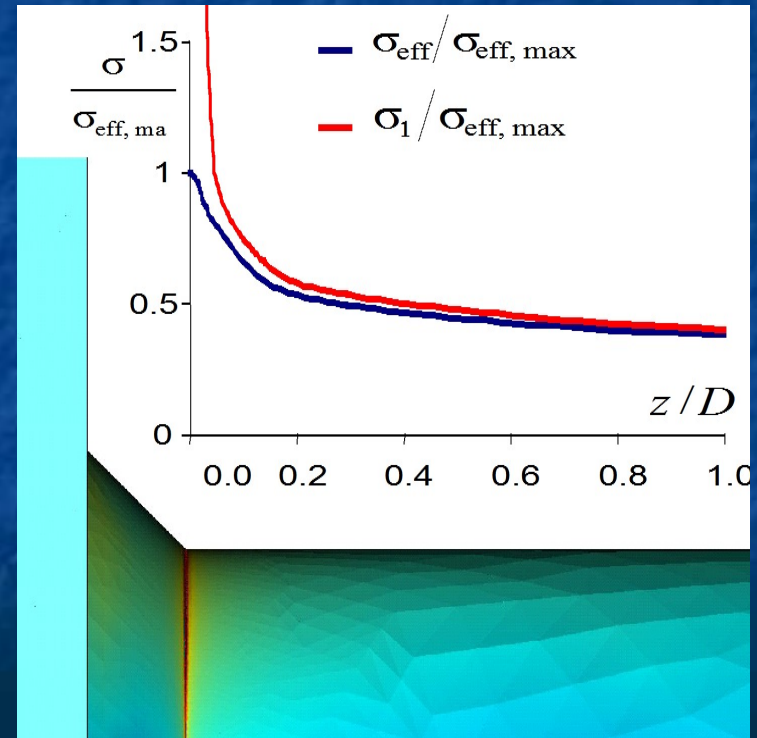
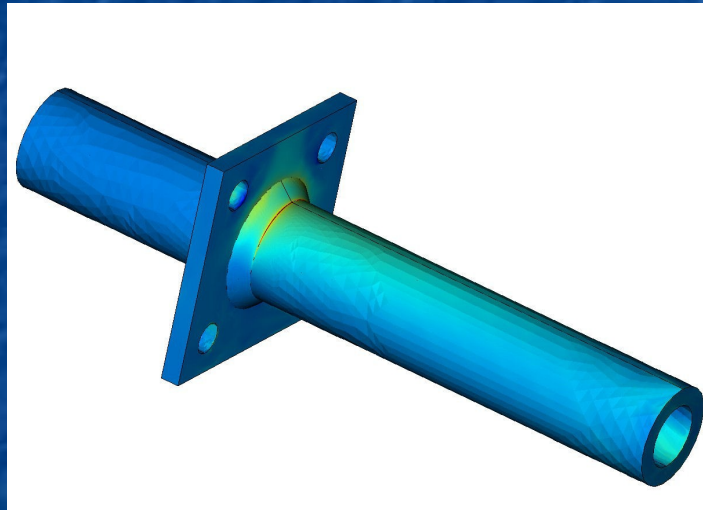


80	<p>A diagram showing a bolted joint on a rectangular plate. The bolt is positioned in the center of the plate. Arrows indicate forces <math>F</math> applied to the bolt and <math>F_r</math> applied to the plate.</p>
80	<p>A diagram showing a bolted joint on a rectangular plate. A spring is placed between the bolt and the plate. Arrows indicate forces <math>F</math> applied to the bolt and <math>F_r</math> applied to the plate.</p>
71	
56	
80	<p>A diagram showing a bolted joint on a rectangular plate. A spring is placed between the bolt and the plate. Arrows indicate forces <math>F</math> applied to the bolt and <math>F_r</math> applied to the plate.</p>
71	

# IL PROBLEMA

## IL METODO DEL GRADIENTE IMPLICITO

RISPONDE ALL'ESIGENZA DI RIPORTARE LA VERIFICA STRUTTURALE AL METODO DELLE TENSIONI AMMISSIBILI ANCHE IN PRESENZA DI SINGOLARITA' TENSIONALI



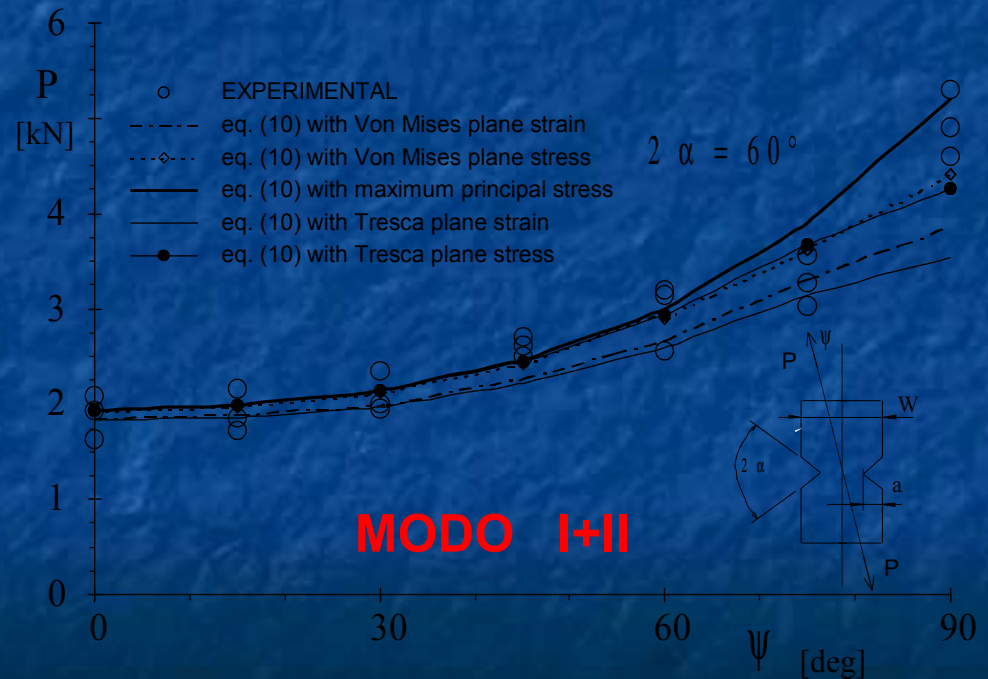
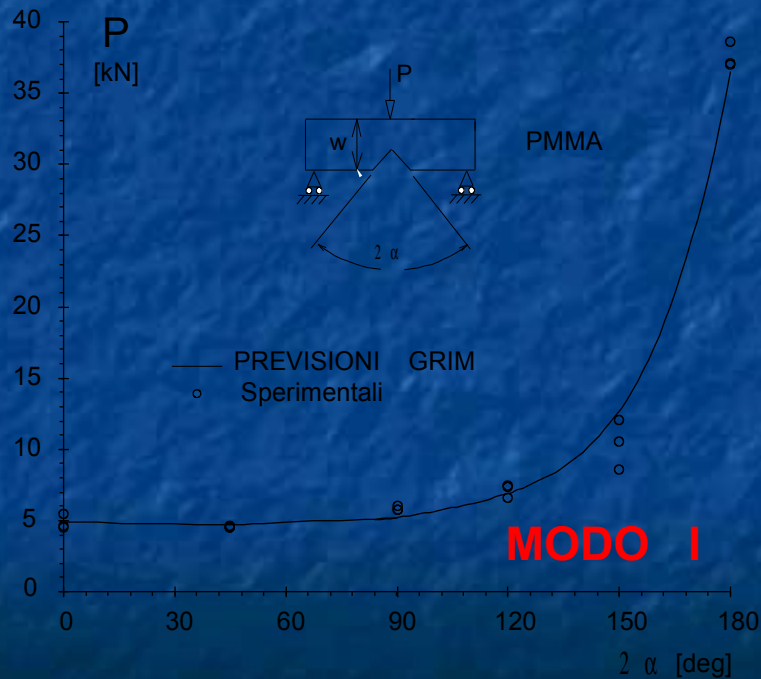
$$\bar{\sigma}_{\text{eff}} \leq \sigma_{\text{limite}}$$

criterio frattura non locale

# CASI AFFRONTATI

## ROTTURA STATICA DI TIPO FRAGILE

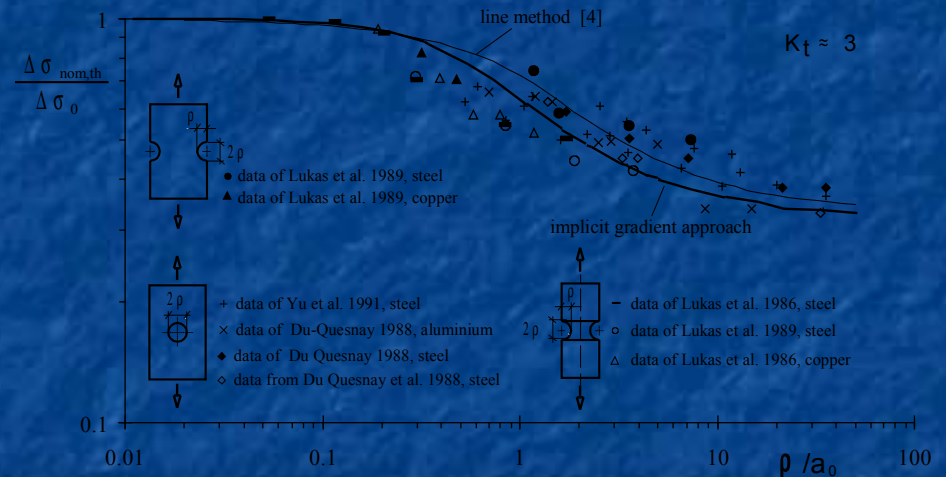
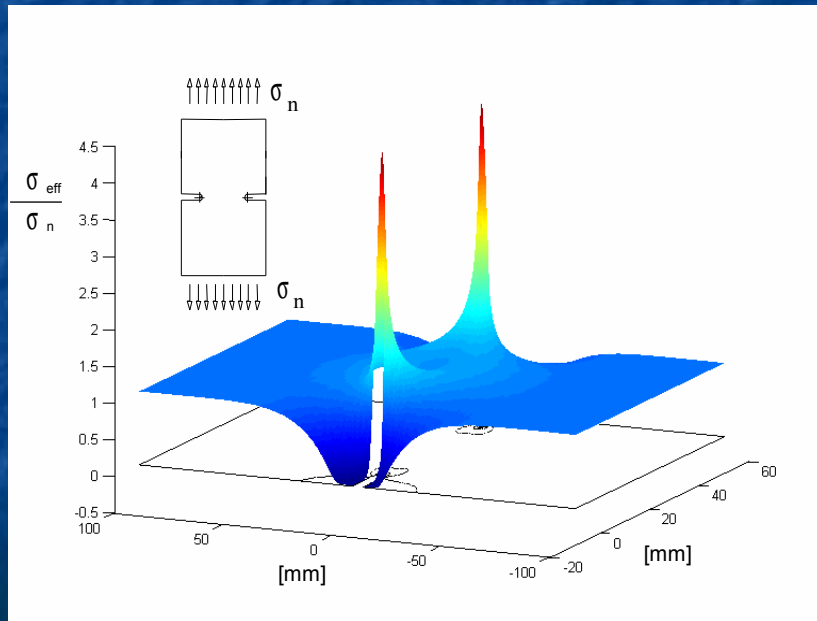
Tovo R., Livieri P., Benvenuti E. An implicit gradient type of static failure criterion for mixed-mode loading. International Journal of Fracture (2006) 141:497–511 E



# CASI AFFRONTATI

## ROTTURA A FATICA DI PROVINI INTAGLIATI

Tovo R., Livieri P. An implicit gradient application to fatigue of complex structures.  
*Engineering Fracture Mechanics*, Volume 75, Issue 7, Pages 1804-1814, May 2008

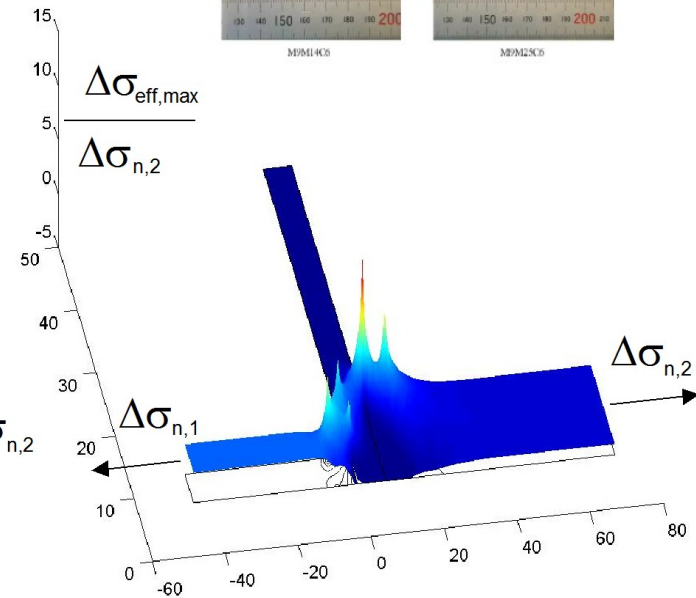
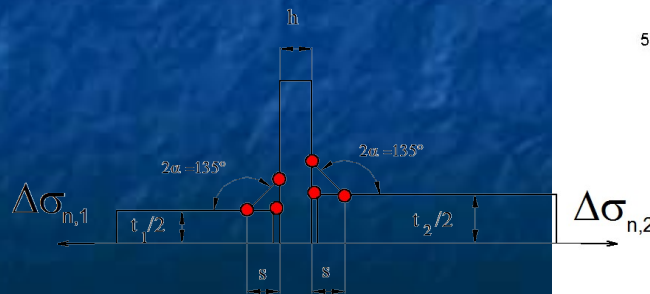
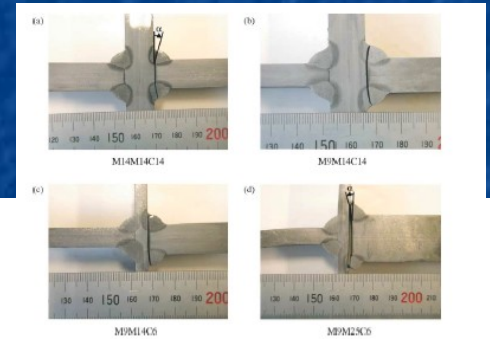
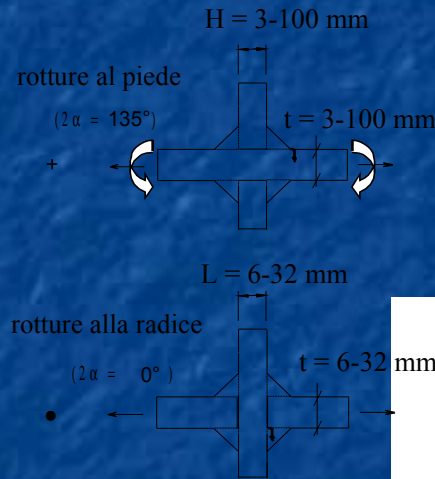
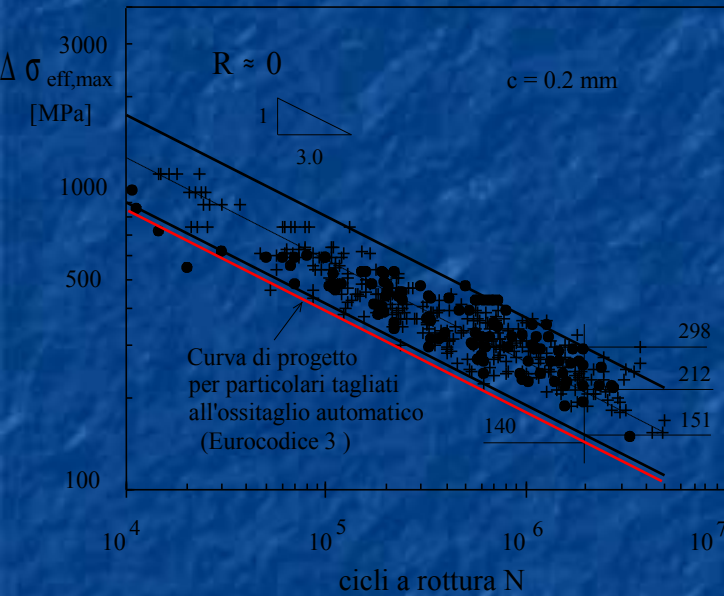


- $\rho$  : raggio di raccordo
- $a_0$  : distanza critica di El Hallad
- $\Delta \sigma_0$  : limite di fatica del provino liscio
- $\Delta \sigma_{nom,th}$  : limite di fatica del provino intagliato

# CASI AFFRONTATI

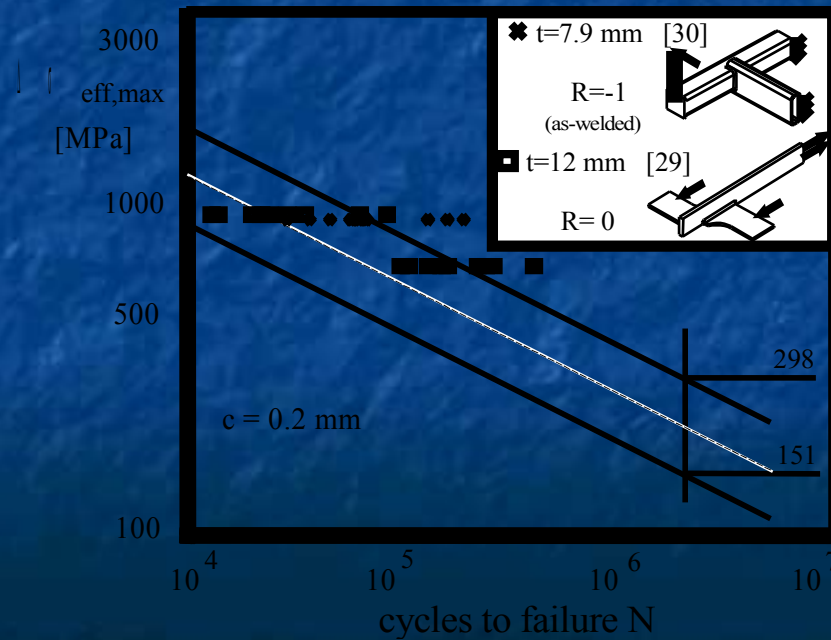
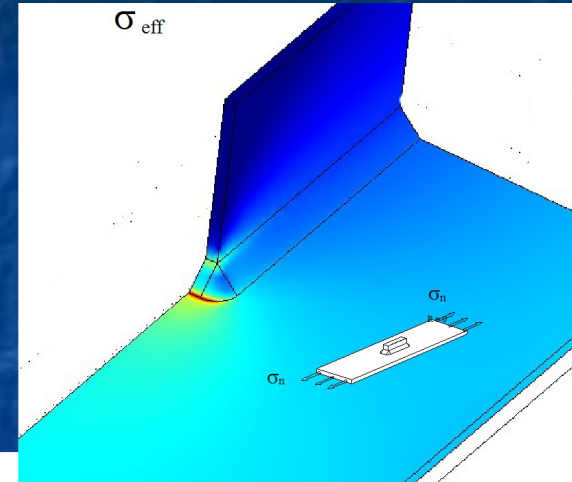
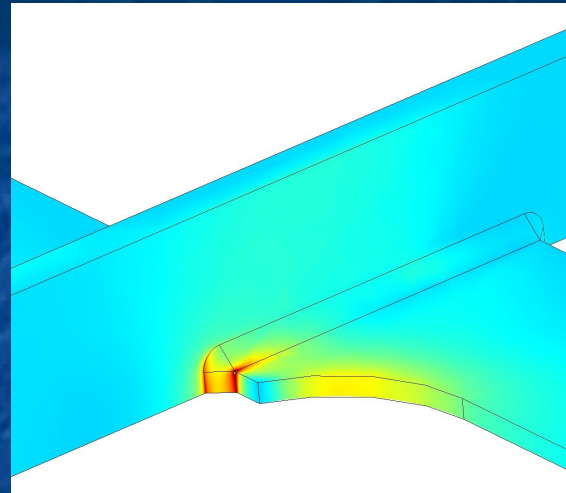
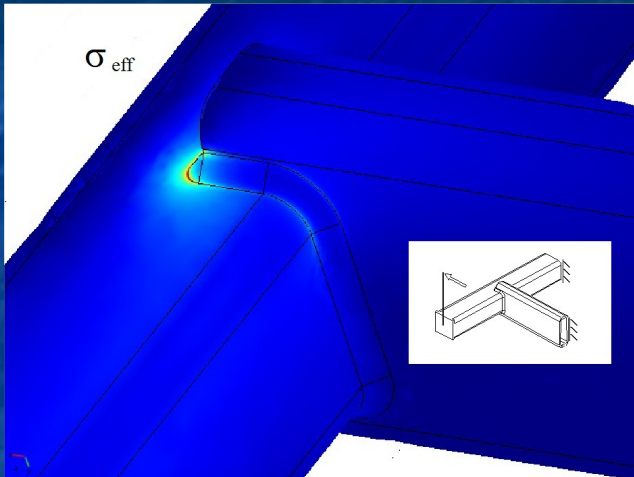
## ROTTURA A FATICA DI GIUNZIONI SALDATE BIDIMENSIONALI

TOVO R., LIVIERI F. An implicit gradient application to fatigue of sharp notches and weldments. *Engineering Fracture Mechanics* 74 (2007) pp. 515–526



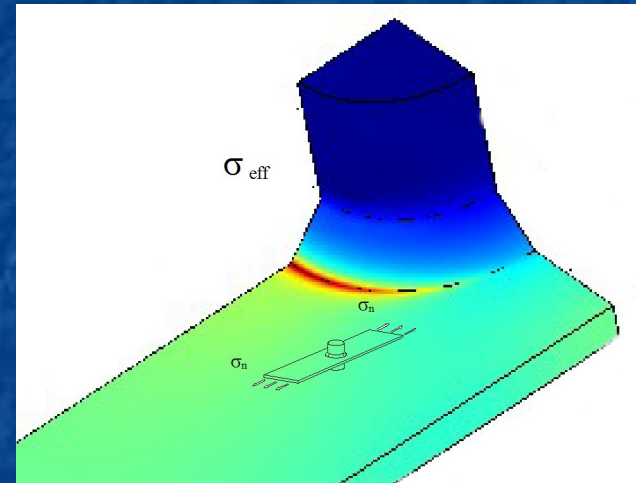
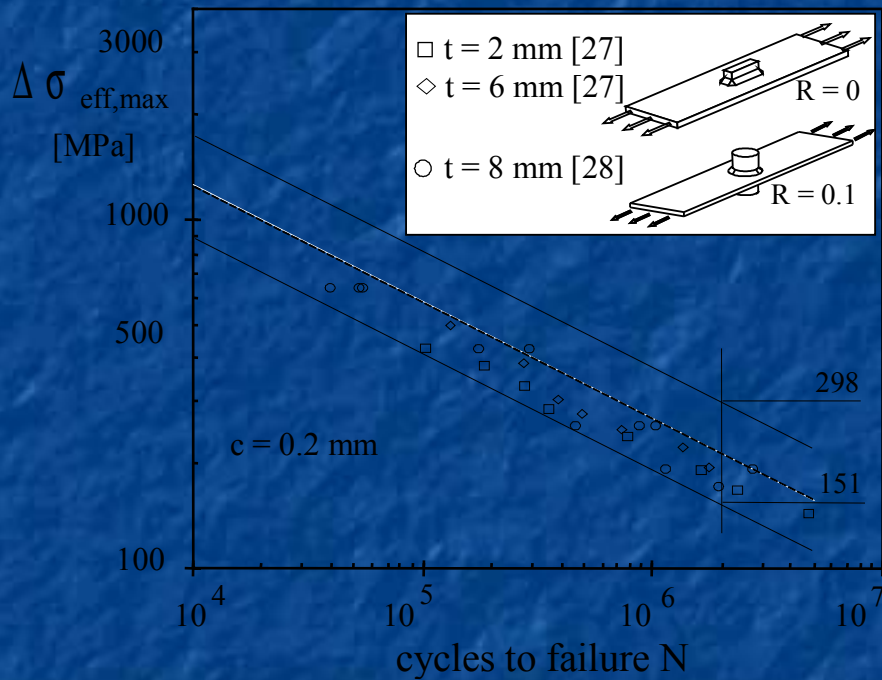
# CASI AFFRONTATI

## ROTTURA A FATICA DI GIUNZIONI SALDATE TRIDIMENSIONALI



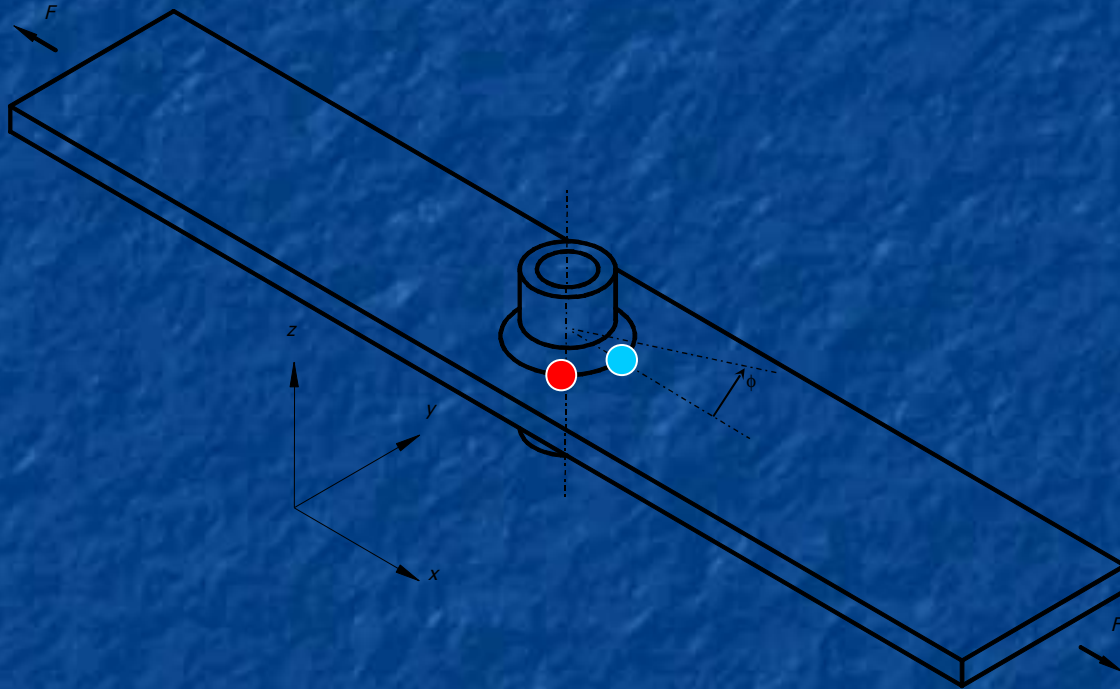
# CASI AFFRONTATI

## ROTTURA A FATICA DI GIUNZIONI SALDATE TRIDIMENSIONALI

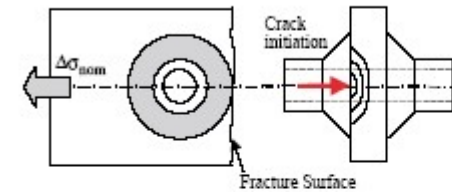


# RISULTATI SPERIMENTALI

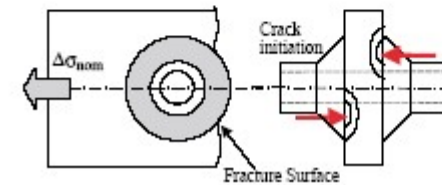
previsione della resistenza a fatica **con**  
**soluzione completamente numerica**



B-Type Joint  $\Delta\sigma_{nom}=90$  MPa,  $N_f=1700 \cdot 10^3$  cycles

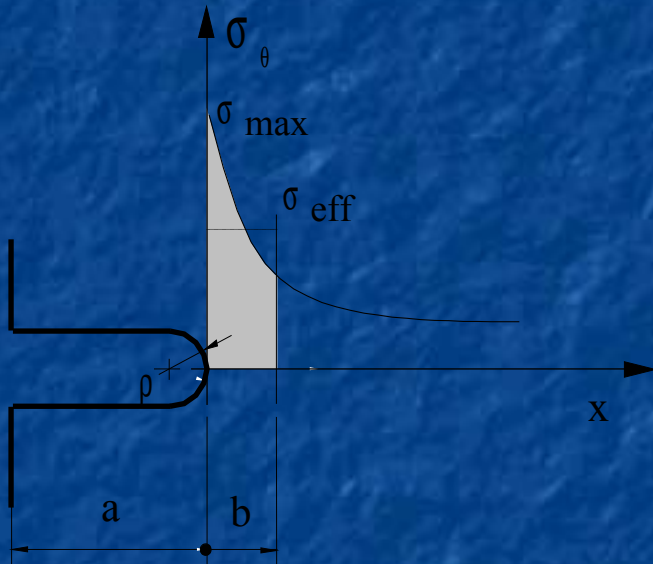


B-Type Joint  $\Delta\sigma_{nom}=200$  MPa,  $N_f=103 \cdot 10^3$  cycles



# FONDAMENTI TEORICI

L'IDEA DELLA MEDIA: NEUBER (1936)



$$\sigma_{\text{eff}} = \frac{1}{b} \int_0^b \sigma_\theta \, dx$$

# FONDAMENTI TEORICI

Utilizzando criteri non locali ispirati ai lavori di Kroener, Eringen, Edelen anni 70'

- Critico ogni punto
- Pesare in funzione della distanza

$$\bar{\sigma}_{eff}(x) = \frac{1}{V_r(x)} \int_V \alpha(x, y) \sigma(y) dy \quad \text{in } V$$

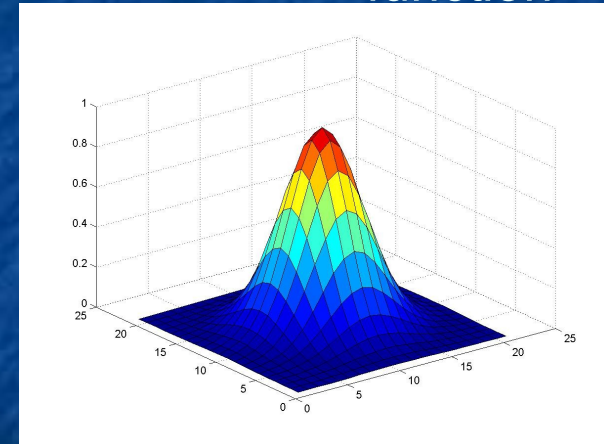
$\bar{\sigma}_{eff}$  stress equivalente non locale

$\sigma$  stress non locale

$\alpha(x, y)$  funzione peso

$$V_r(x) = \int_V \alpha(x, y) dy \quad \text{volume di riferimento}$$

$\alpha(x, y)$  Gauss weight function



Espandendo in serie la tensione non locale si  
perviene all'equazione differenziale

$$\bar{\sigma}_{eff}(x) - c^2 \nabla^2 \bar{\sigma}_{eff}(x) = \sigma(x) \quad \text{in } V$$

$c$  costante legata al materiale  
e allo scalare  $\sigma$

Lo scalare  $\sigma$  (per un assegnato punto)

$$\sigma = f[\underline{\underline{\sigma}}(t)]$$

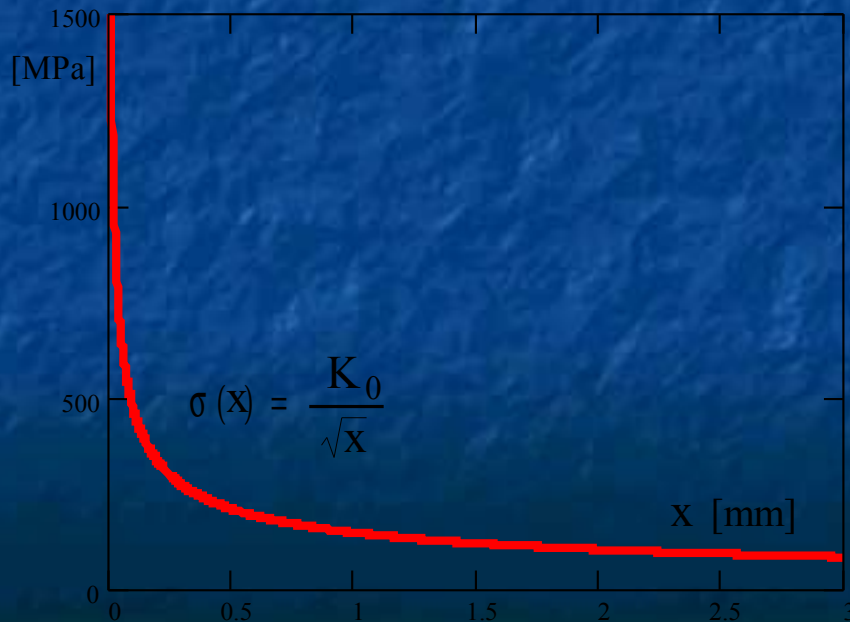
$$\underline{\underline{\sigma}}(t) = \begin{bmatrix} \sigma_x(t) & \tau_{xy}(t) & \tau_{xz}(t) \\ \tau_{yx}(t) & \sigma_y(t) & \tau_{yz}(t) \\ \tau_{zx}(t) & \tau_{zy}(t) & \sigma_z(t) \end{bmatrix}$$

$$\underline{\underline{\sigma}}(t) = \underline{\underline{\sigma}}_d(t) + \sigma_h(t) \cdot I$$

# Soluzioni analitiche di tipo monodimensionale

$$c^2 \frac{d^2 \bar{\sigma}_{eff}}{dx^2} - \bar{\sigma}_{eff} = -\sigma$$

$$\sigma = \frac{K_0}{\sqrt{x}} \quad x \in (0, R)$$

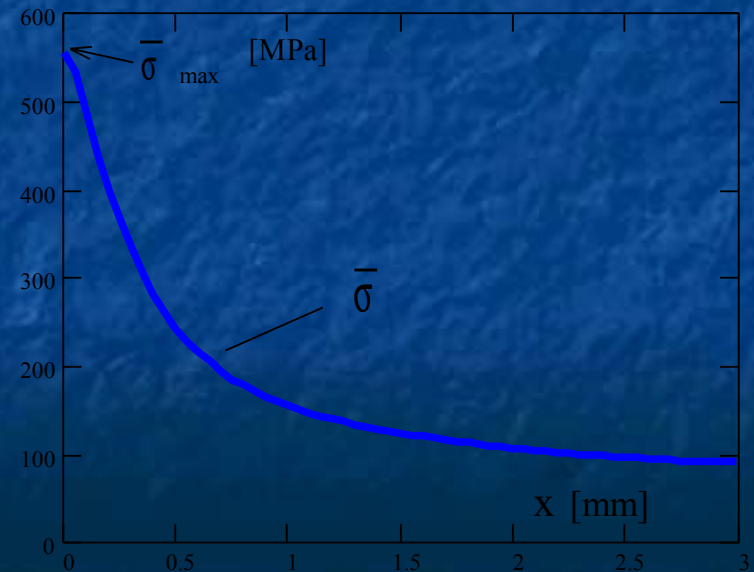
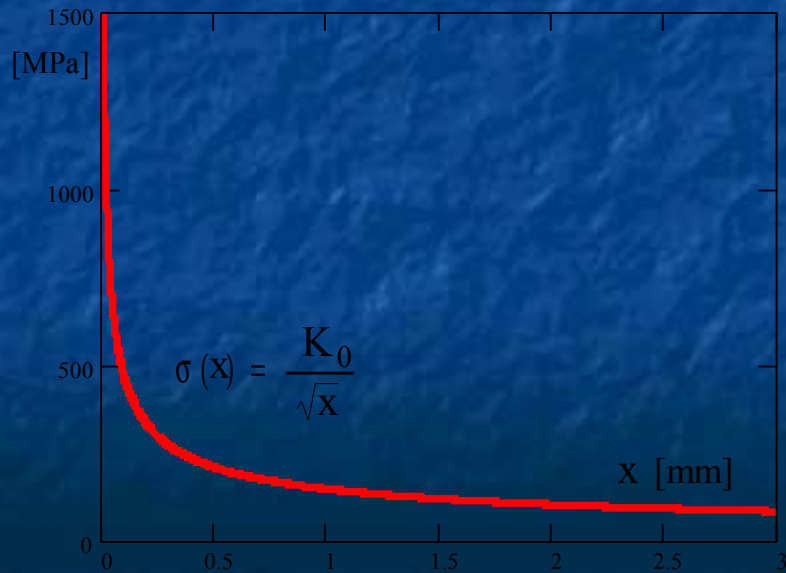


$$\bar{\sigma}_{eff}(x) = \sum_n b_n \cos \frac{n\pi x}{R}$$

# Soluzione Monodimensionale

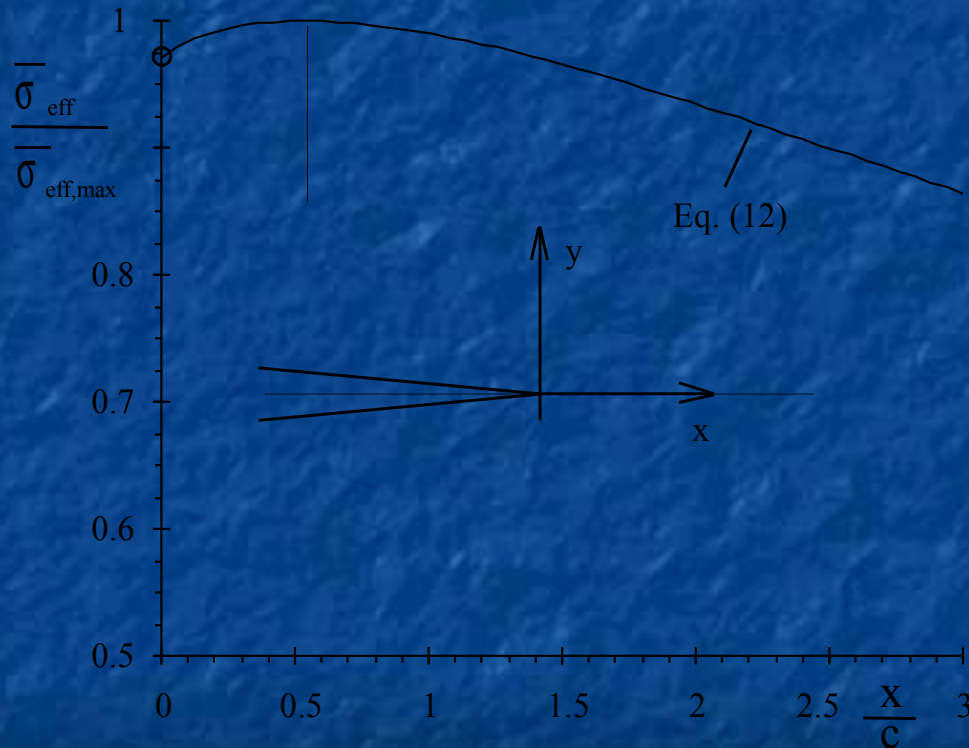
$$b_n = 4 R n \pi K_0 \frac{\sqrt{R} + \sum_{k=0}^{\infty} R^{0.5} (-1)^{k+1} \frac{\pi^{2k+2} n^{2k+2}}{(4k+5)(2k+2)!}}{c^2 n^2 \pi^2 (\cos(n\pi) \sin(n\pi) + n\pi) + R^2 (n\pi + \cos(n\pi) \sin(n\pi))}$$

$$b_0 = \frac{2 K_0}{\sqrt{R}} + \frac{2 K_0}{\sqrt{R}} \sum_{k=0}^{\infty} (-1)^{K+1} \frac{\pi^{2K+2}}{(4k+5)(2k+2)!}$$



# Soluzioni Analitiche di tipo BIDIMENSIONALE

$$\sigma = f[\underline{\underline{\sigma}}(t)] = \sigma_1$$



Cricca

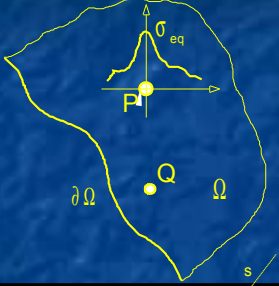
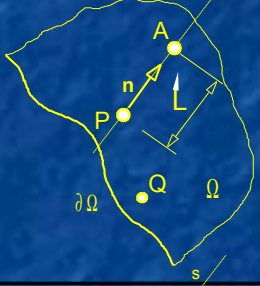
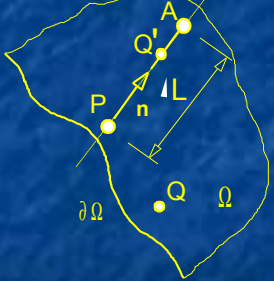

$$\sigma_{\text{eff}}(x) = \frac{K_I}{2\sqrt{2}\pi^{\frac{3}{2}}c^{\frac{3}{2}}} \int_0^{\infty} \int_{-\pi}^{\pi} \sqrt{r} K_0\left(\frac{\rho}{c}\right) f_{\theta} d\theta dr$$

$$\bar{\sigma}_{\text{eff}}(0) = \frac{3}{2} \frac{\Gamma^2\left(\frac{3}{4}\right) K_I}{\pi^{\frac{3}{2}} \sqrt{c}}$$

Intaglio a V

$$\bar{\sigma}_{\text{eff,max}} = 0.44 \frac{\Delta K_{N,1}}{c^{1-\lambda_1}}$$

# METODI DELLE DISTANZE CRITICHE

<p>Peak stress</p>	$P, Q \in \Omega$ $P : \sigma_{eq}(P) = \text{maximum of } \sigma_{eq}$ $\alpha = \delta(P, Q)$	
<p>Point methods</p>	$P, A, Q \in \Omega$ $A \in s :  PA  = L$ $\alpha = \delta(A, Q)$ $P, A, Q, Q' \in \Omega$	
<p>Line methods</p>	$A \in s :  PA  = L \quad Q' \in s : 0 \leq  PQ'  \leq L$ $\alpha(P, Q) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \delta(Q'_k, Q)$	
<p>Area methods</p>	$\Omega' \in \Omega$ $P, Q \in \Omega \quad \alpha = \begin{cases} 1 & \text{if } Q \in \Omega' \\ 0 & \text{if } Q \notin \Omega' \end{cases}$	

# CRITERIO MULTIASSIALE

*A. Cristofori, P. Livieri, R. Tovo, An Application of the Implicit Gradient Method to welded structures under multiaxial fatigue loadings, International Journal of Fatigue*

$$\underline{S} = [S_1 \quad S_2 \quad S_3 \quad S_4 \quad S_5] = \begin{bmatrix} \frac{\sqrt{3}}{2} (\sigma_x - \sigma_h) \\ \frac{1}{2} (\sigma_y - \sigma_z) \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix}$$

Crossland invariant criterion

$$\Delta \sigma_d = \sqrt{3} \Delta S \quad \left\{ \Delta \sigma_d \equiv \Delta \sigma_{\text{von Mises}} \right\}$$

Carichi in fase, R=0

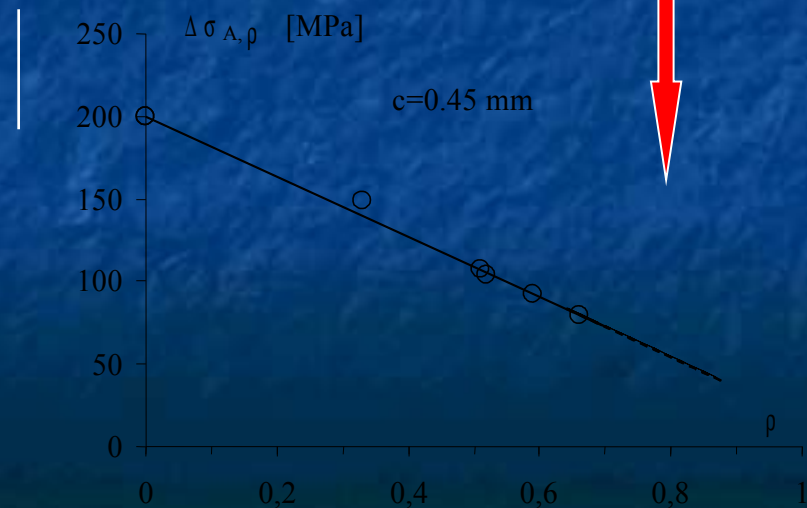
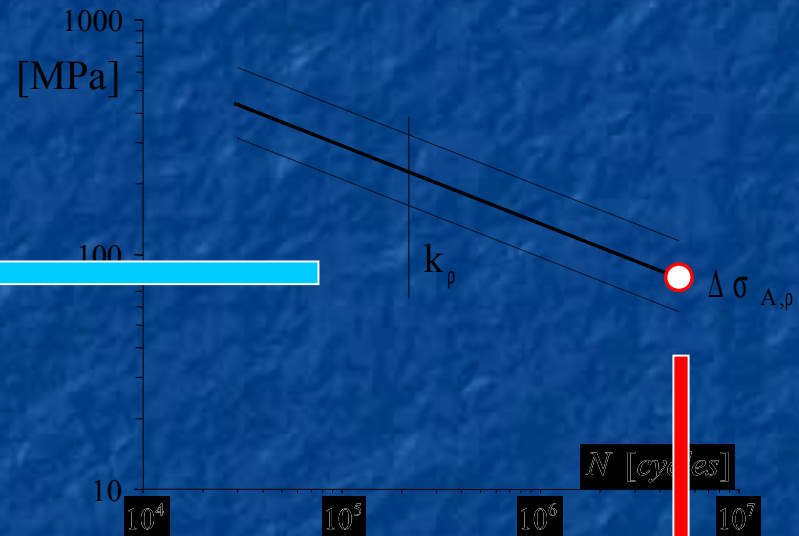
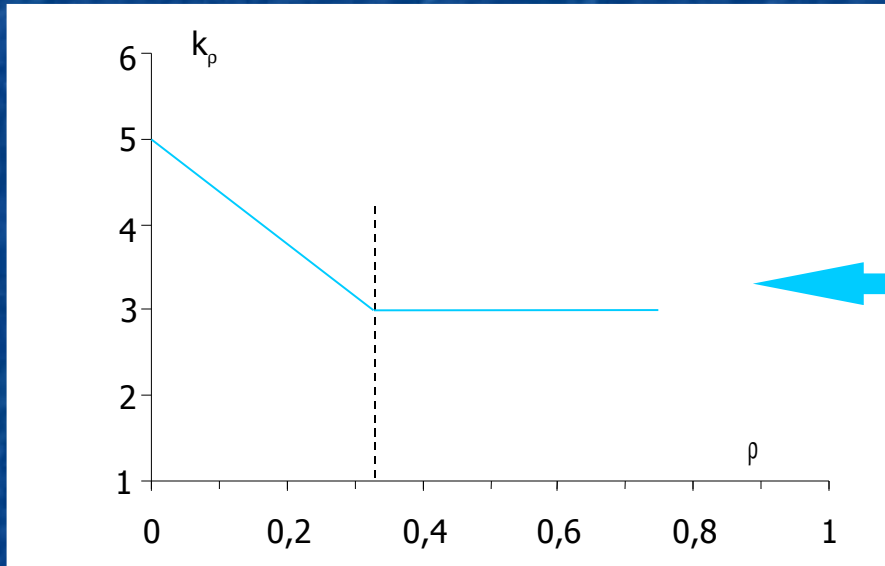
$$\rho = \frac{\Delta \bar{\sigma}_h}{\Delta \bar{\sigma}_d}$$

Parametro di multiassialità

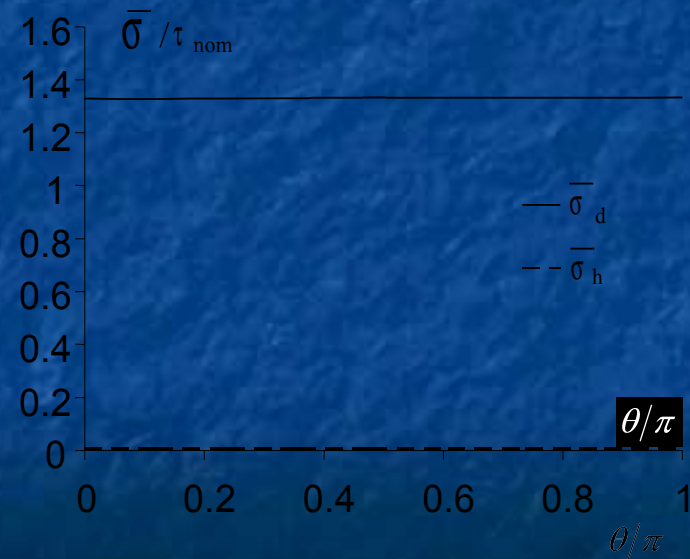
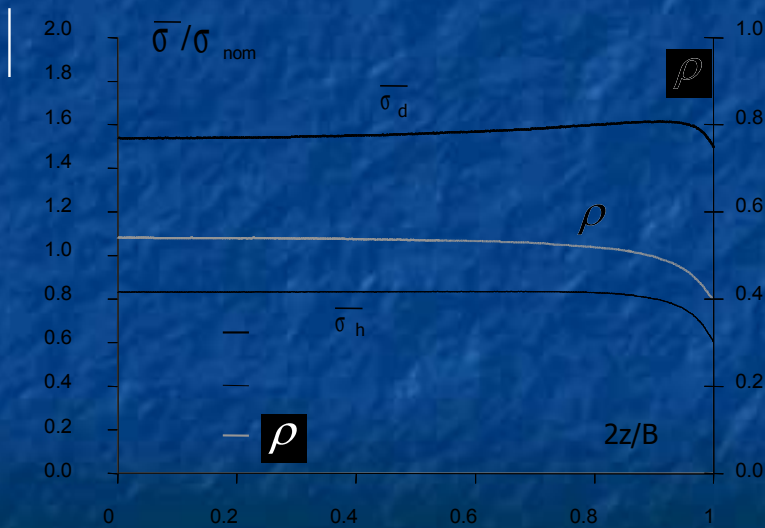
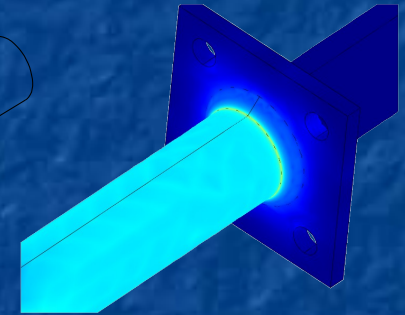
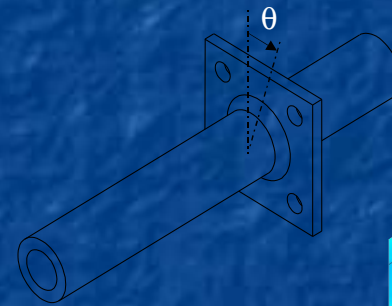
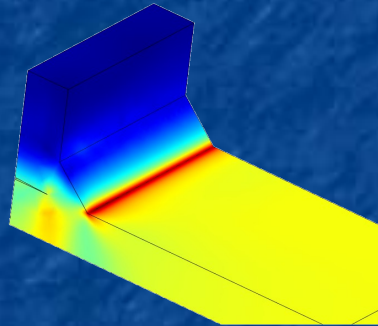
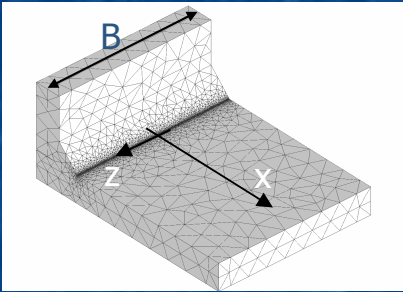
$$N_f = \left[ \frac{\Delta \sigma_{A,p}}{\Delta \bar{\sigma}_d} \right]^{k_p} \cdot N_A$$

Curva di Woehler

# Previsione della resistenza a fatica

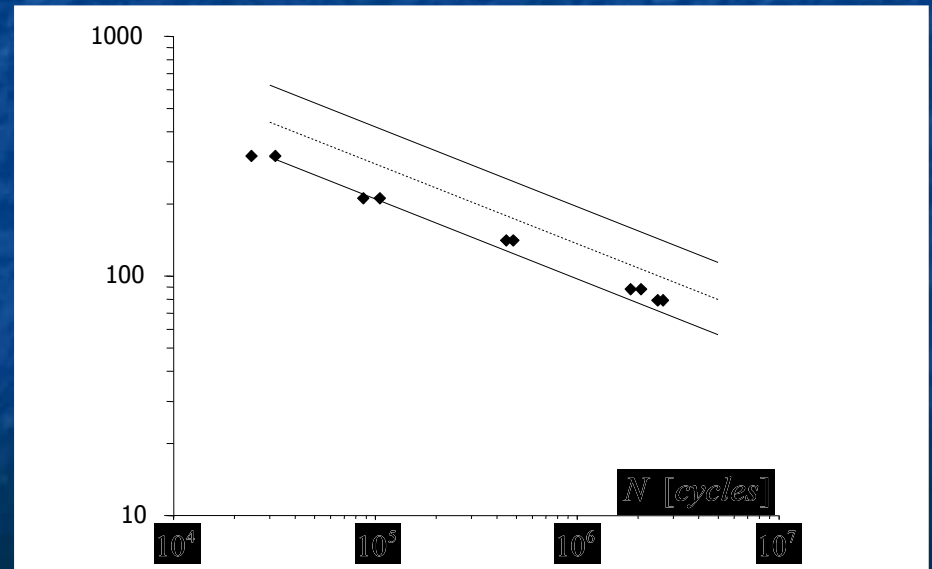
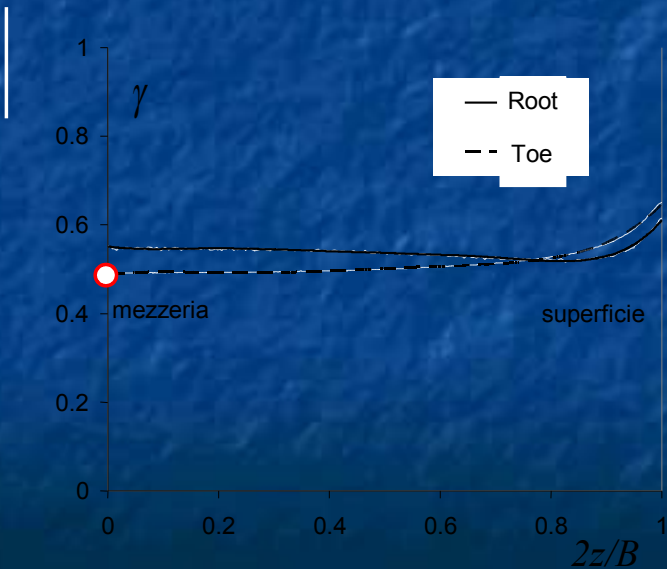
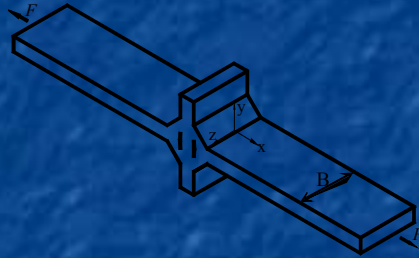


# Parametri di multiassialità



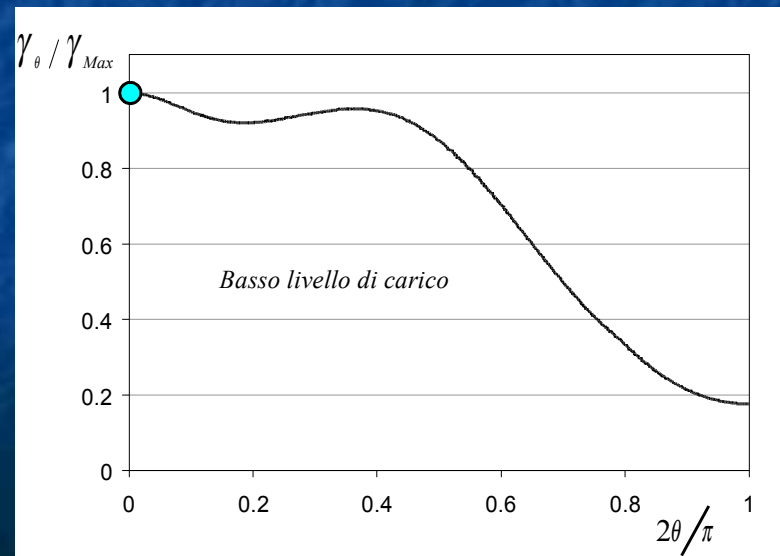
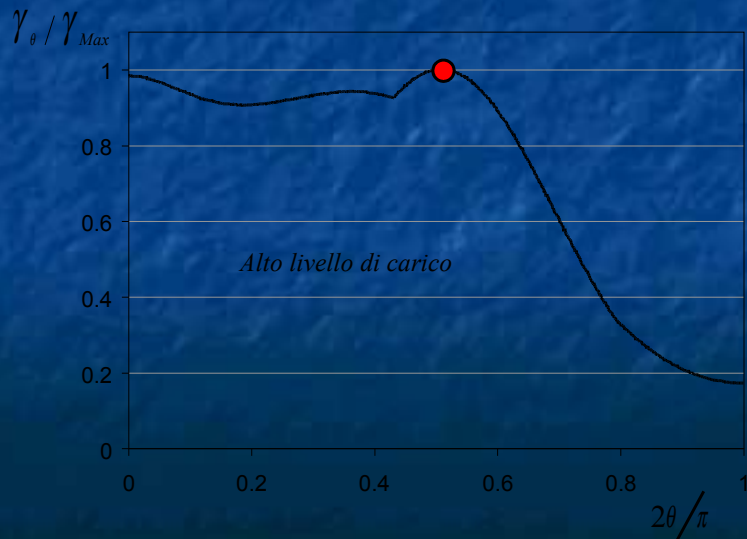
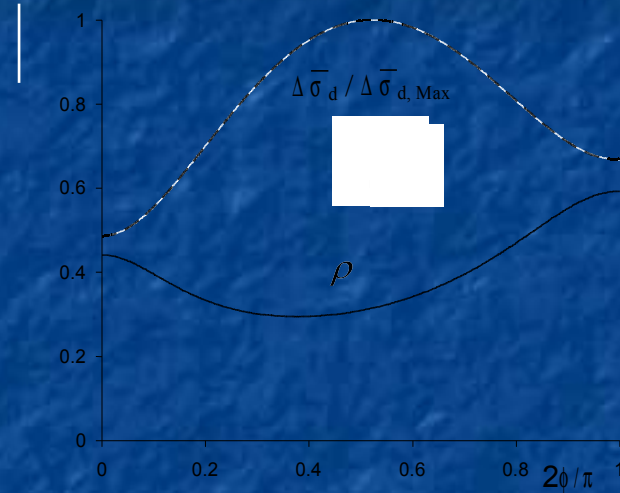
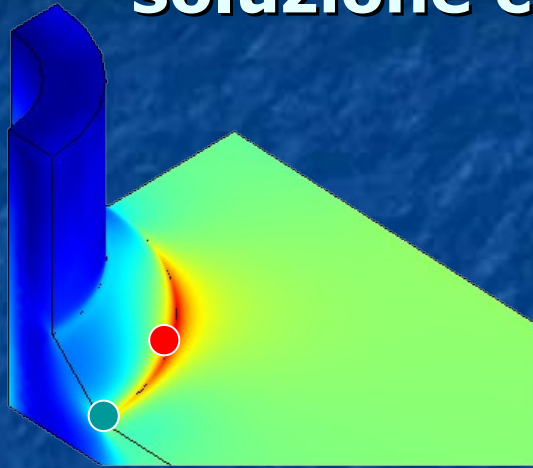
# RISULTATI SPERIMENTALI

previsione della resistenza a fatica **con**  
**soluzione completamente numerica**



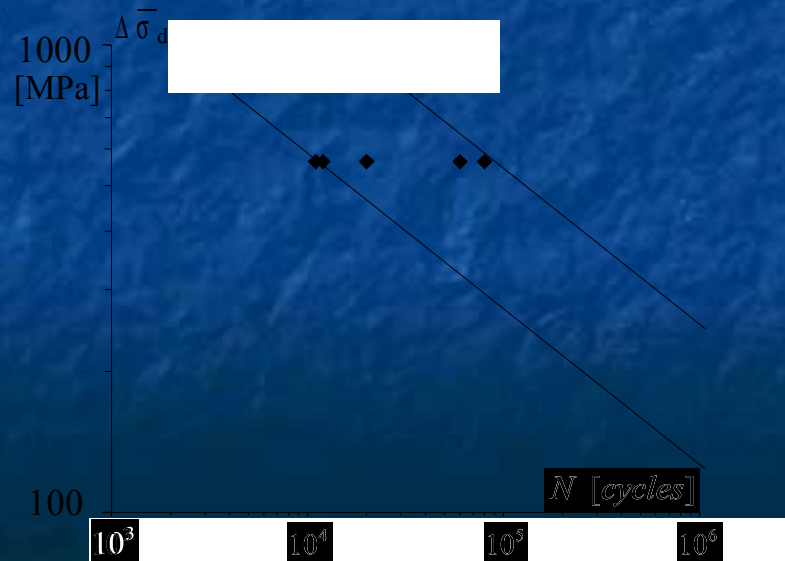
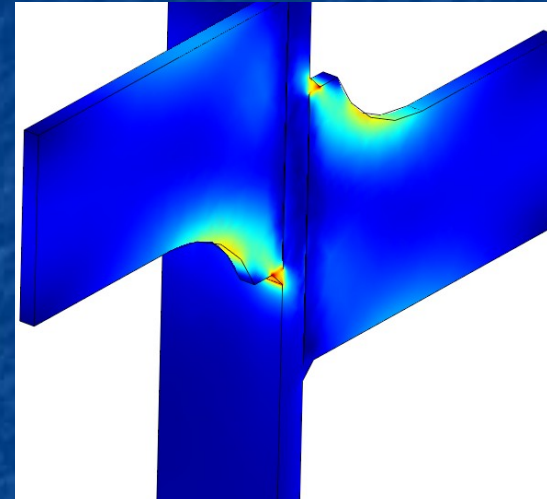
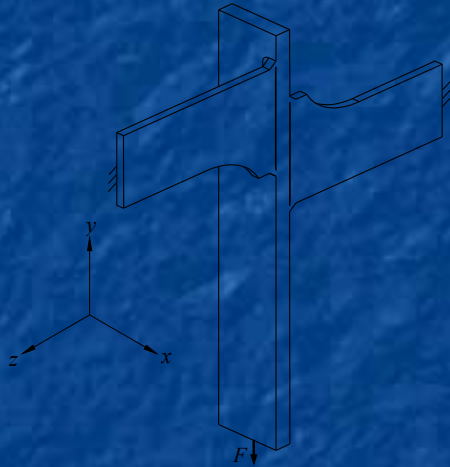
# RISULTATI SPERIMENTALI

previsione della resistenza a fatica **con**  
**soluzione completamente numerica**



# RISULTATI SPERIMENTALI

previsione della resistenza a fatica **con**  
**soluzione completamente numerica**



# CONCLUSIONI

- Con il metodo del gradiente implicito è possibile definire uno **scalare equivalente** da utilizzare per la progettazione a fatica
- Il problema del gradiente di tensione viene trasformato nella **risoluzione di una equazione differenziale** definita sull'intero corpo in esame senza impostare a priori il punto di innesco della frattura
- Il metodo proposto offre il vantaggio di prestarsi per una **soluzione completamente numerica** del calcolo della vita a fatica delle giunzioni saldate complesse.